

QUANTIFYING OCEAN ACOUSTIC ENVIRONMENTAL SENSITIVITY

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1. INTRODUCTION

This paper defines and examines linear, linearized, and nonlinear measures of environmental sensitivity for ocean acoustic propagation that account for realistic uncertainties in various environmental parameters (water-column sound-speed profile and seabed geoacoustic properties). Simple interpretations of sensitivity are typically based on the implicit assumption of a linear relationship between parameter sensitivity and parameter uncertainty. This assumption is examined by comparing the three sensitivity measures over a range of parameter uncertainties about the actual assumed environmental uncertainty. Sensitivity range and depth dependencies are illustrated for realistic geoacoustic uncertainties and oceanographic variability of the sound-speed profile.

2. THEORY

The interpretation of sensitivity is well defined for linear problems, which provide a basis for addressing nonlinear problems such as ocean acoustic propagation. For a linear problem, the change in datum d_i due a change δm_j to the j th environmental model parameter m_j is

$$\delta d_i^j = d_i(m_j + \delta m_j) - d_i(m_j) = \frac{\partial d_i}{\partial m_j} \delta m_j. \quad (1)$$

Equation (1) indicates how uncertainties transfer from parameters to data for a linear problem: If the uncertainty for parameter m_j is Gaussian distributed with standard deviation σ_j , then the corresponding uncertainty for datum d_i is also Gaussian with standard deviation $\partial d_i / \partial m_j \sigma_j$; i.e., sensitivities scale directly with parameter uncertainties. A linear sensitivity measure that is independent of data/parameter scales and units is defined

$$S_{ij} = \frac{|\delta d_i(\delta m_j = \sigma_j)|}{|d_i|}. \quad (2)$$

Equation (2) represents the ratio of the standard deviation of the i th datum to its expected value and is equivalent to the coefficient of variation, a quantity commonly used for comparing the variability of potentially disparate quantities.

The above concepts can be extended to weakly nonlinear problems. Expanding the data functional for a parameter perturbation about the background model and neglecting second-order terms leads to an approximate local linear relationship between data and parameter perturbations which can be used in a linearized sensitivity measure; for improved accuracy, a two-sided average can be defined

$$S_{ij} = \frac{1}{2} \left[\frac{|d_i(m_j + \delta\sigma_j) - d_i(m_j)|}{|d_j|} + \frac{|d_i(m_j - \delta\sigma_j) - d_i(m_j)|}{|d_j|} \right] \quad (3)$$

To address nonlinearity explicitly, the full data uncertainty distribution can be sampled using a Monte Carlo approach to draw random model perturbations δm_j from a Gaussian distribution with zero mean and standard deviation σ_j , and computing the corresponding data perturbation for each sample. A nonlinear sensitivity measure is based on the root-mean-square average perturbation

$$S_{ij} = \frac{\left[\left\langle \left[d_i(m_j + \delta m_j) - d_i(m_j) \right]^2 \right\rangle \right]^{1/2}}{|d_i|} \quad (4)$$

For a linear problem, the linear, linearized, and nonlinear sensitivity measures are identical. Due to the highly variable nature of propagating acoustic fields, data sensitivities must be spatially averaged, as indicated in the following section, to obtain a stable, representative sensitivity measure [1].

The linearity of parameter sensitivities can be examined by comparing the linear, linearized, and nonlinear sensitivity measures over a range of parameter standard deviations σ_j about the actual environmental uncertainty.

3. EXAMPLES

This section provides examples of the sensitivity analysis outlined above. The environmental parameters and uncertainties are based on the Malta Plateau, a well-studied region of the Mediterranean Sea. The environmental model, illustrated in Fig. 1, is comprised of a 131-m water column

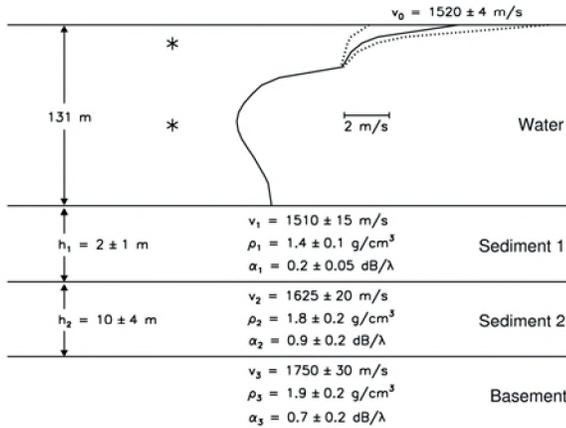


Fig. 1. Malta Plateau environment, including assumed values and standard deviations for environmental parameters. Asterisks indicate source depths.

over a three-layer seabed. The ocean sound-speed profile (SSP) includes a strong negative gradient in the near-surface waters and a weak sound channel with its axis near mid-water depth. The three seabed layers are characterized by sound speeds v_1 , v_2 , v_3 , densities ρ_1 , ρ_2 , ρ_3 , and attenuation coefficients α_1 , α_2 , α_3 . The upper two sediment layers are of thicknesses h_1 and h_2 (the basement layer is semi-infinite). Geoacoustic and SSP parameters values and uncertainties representative of the Malta Plateau region are given in Fig. 1. The SSP uncertainty is taken to represent oceanographic variability due to surface heating/cooling and wind mixing, with the effects decaying exponentially with depth over the top 30 m, as shown in Fig. 1. This variability is represented by the standard deviation of the surface sound speed v_0 .

Fig. 2 shows the three sensitivity measures for all environmental parameters averaged over 0–30 m receiver depth and 0–20 km range for source depths of 15 and 65 m. Sensitivities for the different parameters vary over almost six orders of magnitude, with the most sensitive parameters being those of the upper seabed layer and SSP, followed by the second and then third seabed layers. The sensitivity results for the two source depths in Fig. 4 are generally similar, with the largest difference for the SSP parameter v_0 . In all cases the linearized estimate provides a better approximation to the true nonlinear sensitivity measure.

Figure 3 illustrates the three sensitivities for selected parameters as a function of range and depth, with the parameter standard deviation fixed at the assumed environmental uncertainties for the Malta Plateau region (Fig. 1). For the three geoacoustic parameters v_1 , v_2 , and h_1 , the source is at 65-m depth near the sound-channel axis, while for the SSP parameter v_0 , the source is at 15-m depth within the variable near-surface layer. The sensitivity to the second sediment layer sound speed v_2 includes high values at short ranges, as steep propagation paths (which attenuate with range) interact significantly with the deeper layer. The

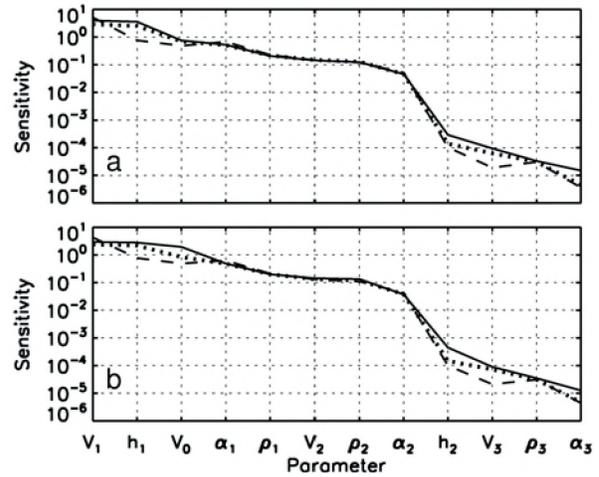


Fig. 2. Parameter sensitivities averaged over receiver depths of 0–30 m for source depths of (a) 65 m and (b) 15 m. Nonlinear, linearized, and linear sensitivity measures are indicated by solid, dotted, and dashed lines, respectively.

highest sensitivity for the SSP parameter v_0 is confined to depths over which the SSP varies. The agreement of the linearized and linear sensitivities with the nonlinear sensitivity in Fig. 3 is good for parameter v_1 and excellent for v_2 , with generally better agreement for the linearized measure. The linearized sensitivity is in reasonable agreement with the nonlinear sensitivity for parameter h_1 , but in poor agreement for v_0 . The linear sensitivity differs substantially from the nonlinear sensitivity for h_1 and v_0 .

REFERENCES

- [1] R. T. Kessel, “A mode-based measure of field sensitivity to geoacoustic parameters in weakly range-dependent environments,” *J. Acoust. Soc. Am.*, **105**, 122–129 (1999).

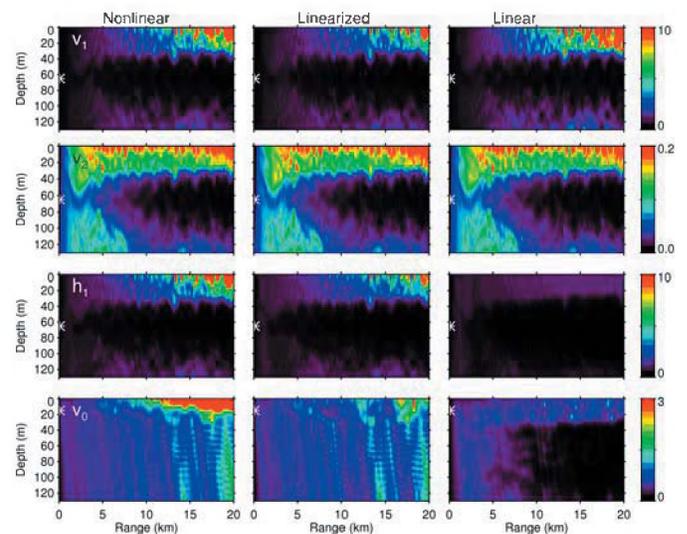


Fig. 3. Sensitivity sections for parameters v_1 , v_2 , h_1 , and v_0 (rows) and nonlinear, linearized, and linear sensitivity measures (columns). Source depth indicated by asterisks.