On Estimating Sound Power From A Few Single Point Measurements

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1. Introduction

Even under controlled conditions the measurement of sound power emitted by a stationary source can present difficulties. In the field, these are often augmented by constraints that preclude measuring at a sufficiently large number of points. This note describes the nature of a sound field generated by a broad-band source located above a reflecting surface and discusses some of the implications regarding the estimation of sound power levels from a small number of field measurements.

2. Mathematical Model

A simple source radiating into an unbounded space, induces a spherically symmetric acoustic field about the centroid of the source: $p(r,t)= A/r\, g(t-r/c)$. In the far-field the flux of acoustic energy $\langle p^2\rangle/\rho c$, when integrated over a sphere of radius r, recovers the sound power W. A^2 takes the value $\rho cW/4\pi$ if $\langle g^2\rangle =1$. An refelcting plane h_s meters below the source confines all the acoustic energy to the 'upper half space'. This is equivalent to the field of a source and a 'mirror image' of identical strength (figure 1)

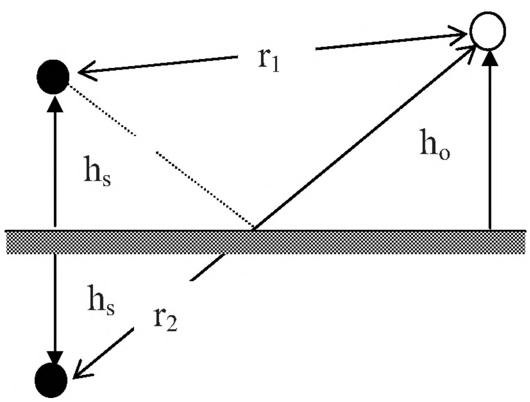

Figure 1. Simple source above a reflecting plane

The mean-square sound pressure is composed of three terms, two of which are contributions from the direct and reflected waves. The third term describes the interaction of the two wave fields:

$$\langle p^2(r)\rangle = A^2\,[1/r_1^2+1/r_2^2+2\, R_{gg}((r_2-r_1)/c)/(r_1 r_2)\,] \qquad [1]$$

The differential time delay in the argument of $R_{gg}(\tau)$ plays an important role, even though it frequently ignored. Many field measurements are performed with 1/3 octave band analyzers. The autocorrelation function of the band-pass filtered signal can be approximated without a priori knowledge of the power spectral density (PSD) of the function g(t), provided that there are no pure tones and that the PSD is smooth. The band-limited PSD is modeled with a series of unit steps (H(f)):

$$\Phi_{gg}(f)= \Phi_o\{H(f+f_2)-H(f-f_2)-[(H(f+f_1)-H(f-f_1)]\} \qquad [2]$$

This idealized process differs little from real band-limited systems. It is well known that the power spectral density $\Phi_{gg}(f)$ and the autocorrelation $R_{gg}(\tau)$ are Fourier Transform pairs. The latter resembling a 'cosine burst' centered at $\tau = 0$:

$$R_{gg}(\tau) = \cos\,[\pi(f_2+f_1)\tau]\, \frac{\sin \pi(f_2-f_1)\tau}{\pi(f_2-f_1)\tau} \qquad [3]$$

For a 1/3 octave filter the 'corner frequencies' are defined by in terms of the centre frequency f_o: $f_2=\alpha f_o$, $f1=f_o/\alpha$, where $\alpha=2^{(1/6)}$. The typical response time-scale is $T \sim(f_2-f_1)^{-1}$.

The broad-band nature of the noise does not preclude wave interference. Figure 2 illustrates the distribution of mean square pressure in the 500 Hz 1/3 octave band for a source 0.5 m above the reflecting plane.

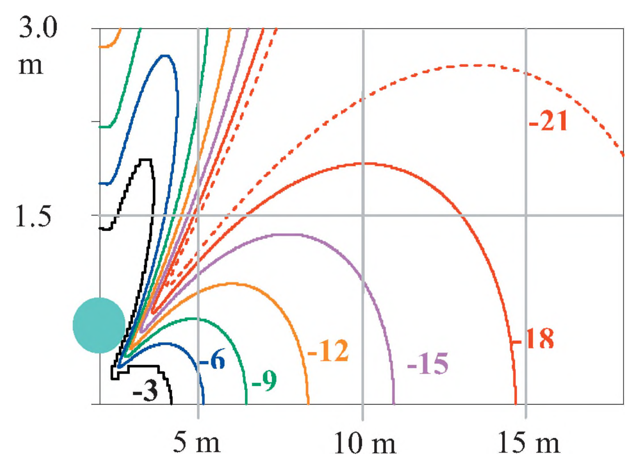

Figure 2 Isobars (in -3dB increments) for the 500 Hz 1/3 octave band levels from a 'broad-band source' located 0.5 m above reflecting plane

The sound field is rather more complex than the hemi-spherical spreading described by the ubiquitous formula:

$$\langle p^2(r)\rangle = 2A^2/r_1^2 \;\text{ or : SPL= Lw - 10log}(2\pi r_1) \qquad [4]$$

3. Estimating Sound Power

It is common practise to 'estimate' sound power levels of sources by one or two 'spot-checks'. The ratio of the estimated sound power [based on eq. 4] and the true sound power is

$$W_{est}/W_{act} = \{0.5[1+r_1^2/r_2^2] + (r_1/r_2)R_{gg}[(r_2-r_1)/c]\} \qquad [5]$$

Whenever the auto-correlation function is negative, the estimated power is smaller than the actual power. Figure 3 illustrates this for a source height of 0.5 m and an observer height of 1.5 m. The 1/3 octave predictions have been converted to octave levels: changing the bandwidth does not eliminate the effect.

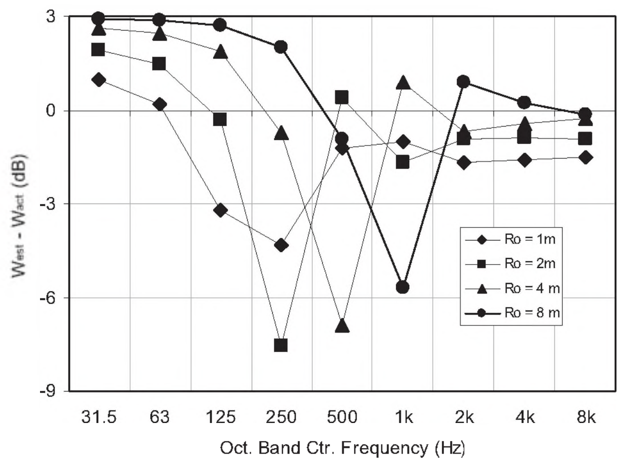

Figure 3 Deviation in sound power estimate (in octave bands) for a simple 'broad-band' source (500 Hz octave band), Ro =distance from the source

At low frequencies the sound power levels are over-estimated. This is because the relative phases of direct and reflected waves differ only by a small fraction of a wavelength. At high frequencies the relative error is small. For moderate source heights the worst-case errors occur in the mid-frequency range.

4. Estimating Sound Power using Measurements conforming to ISO 3744

When more measurement locations are included, one might expect these errors to diminish. The ISO 3744 standard prescribes measurements at selected field points. The hemispherical scheme, based on 10 field points is discussed here. It is found that the sound power estimates based on such measurements exhibit bias errors. These deviations arise irrespective of other 'random errors'.

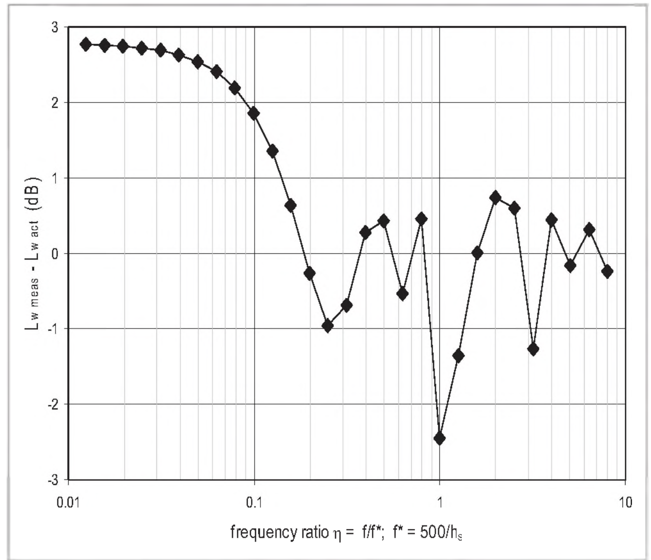

Figure 4 Bias errors for 1/3 octave sound power levels 'measured' according to ISO 3744

Whenever the radius of the (virtual) measurement surface is greater than $4h_s$, the bias error is self-similar in f/f^*; $f^*=500/h_s$. Low frequencies have a positive bias: sound power levels are over-estimated. The bias is small for mid and high frequencies, with some bands showing bias errors of the order of 2 to 3 dB.

5. Summary

Certain features of sound pressure measurements over reflective ground have been examined for compact sources radiating 'broad-band' noise. The formalism uses the method of images and treats the measurement by 1/3 octave band filters in a realistic manner. Even though the sources are assumed to radiate broadband noise, wave effects cannot be ignored. The common 3 dB correction for hemispherical spreading leads to errors that are of the order of 6 dB at mid-frequencies (125 Hz to 1 kHz). Fortunately, these bias errors tend not impact estimates of A weighted sound power levels.

6. References

Beranek, L.L.; Noise and Vibration Control, McGraw Hill, 1971

Anon. Acoustics-Determination Of Sound Power Levels Of Noise Sources Using Sound Pressure—Engineering Method In An Essentially Free Field Over A Reflecting Plane ISO-3744:1994(E)