HIGH FREQUENCY BROADBAND ACOUSTIC CHANNEL ESTIMATION ERROR ANALYSIS FOR ST. MARGARET’S BAY DURING THE UNET06 DEMONSTRATIONS.

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1. INTRODUCTION

We are interested in assessing the adversity of a shallow water acoustic channel to coherent communications. The shallow water channel presents a particularly difficult environment for high rate acoustic communications as channel conditions can change dramatically both spatially from site to site and temporally over durations as short as hours. Shallow water coastal environments can range from very calm with relatively coherent acoustic response functions to severely doubly spread environments where multi-path delay is coupled with path dependent Doppler. Doppler spreading is imparted by the temporal dynamics of the water column along with wind driven surface wave motion. Platform motion imparts severe temporal variations for paths that interact with a rough bottom. Each of these processes impact each acoustic path, with its individual launching and arrival angle, differently. The net effect of this delay spread and rapid temporal variation is that communication receivers must model the acoustic channel accurately in order to efficiently decode the sent data. It is the goal of this work to develop a computationally efficient estimator of the channel response function that includes the innovation variance associated with the temporal fluctuations of the channel and apply these to data collected in St. Margaret’s Bay during the Unet06 demonstrations.

1.1 Experiment design

During the 2006 Underwater networking experiment (Unet06) DRDC-Atlantic with the Naval Research Laboratory conducted a number of high frequency broadband channel probe experiments designed to characterize the underwater acoustic channels adversity to communications. Figure 1 depicts the experiment layout. The DRDC multimode pipe projector (MMPP) provided a 185 dB // 1mPa@1m source level at 44 kHz center frequency and was allowed to drift approximately 2 meters from the bottom in 70m of water at ranges between 300 m and 1 km from the NRL ACDS 8-element vertical receiver array. The receiver array was placed approximately mid water column and was sampled at 160 ksp’s. The doubly spread acoustic channel is estimated from the recorded data sets by an augmented Kalman recursion to be described.

1.2 Model of Acoustic Response Dynamics

The posterior covariance of the acoustic channel is a function of the data and the innovation variance of the Gauss Markov response process. To compute the channel estimate and covariance function this latent innovation variance must be estimated. We augment, by an empirical Bayes approach [1], the Kalman recursion with a point estimate of the innovation variance to improve estimation of the acoustic response function. The underwater acoustic response function \( h_t \) is modeled as Gauss-Markov on the interval of observation, that is \( h_t | h_{-D}, q : N_h(Ah_{-D}, qI_M) \), where \( N_x(mS) = |2\pi S|^{-1/2}\exp\{x- m’S^{-1}(x- m)/2\} \) and \( x | y : p_x(y) \) denotes the density of \( x \) given \( y \). By the form of the innovation covariance we have assumed that the innovation process is uncorrelated and invariant to path delay. The background noise at the \( k^{th} \) receive element is assumed to be spatially uncorrelated, stationary, but with temporal covariance \( S_k \). The received data segment at the \( k^{th} \) hydrophone, over the \( t^{th} \) time interval is \( r_{k,t} \), and distributed as \( r_{k,t} | h_t, s_t, a_t : N_r(S_t h_t, S_k) \). Here \( S_t \) is the convolution matrix formed from the source signal vector \( s_t \), the \( t^{th} \) segment of the source signal, dilated according to \( s_k(t) = \sqrt{a(t)} s(a(t)) \), where \( a(t) \) represents an estimate among all paths and receive elements of the time varying, dilated time index.

1.3. Source signal dilation estimation

The dilation process is dominated by source-receiver platform motion in scenarios where either or both are not rigidly fixed and decoupled from the surface [2,3].
For a fixed receiver array under downward refracting conditions with source drifting near the bottom source motion is the greatest cause of imparted dilation. Estimates of the dilation process by the method described in [1] are presented in Figure 2. The source held from a cable to a surface vessel imparts a noticeable time varying dilation. The accelerations are over 1m/sec$^2$ as depicted.

2. **Motivation for estimating parameters of the Gauss-Markov model**

Under the Gauss-Markov assumption on the acoustic Green’s function with known innovation variance $q$ it follows that $h_t |q, r,e : \mathcal{N}(\hat{h}_t, P_t(q))$ where $r$ represents all data preceding the time of estimation,

\[
\hat{h}_t = (I - G_tS_t)\hat{h}_t + G_t R_t; P_t = (I - G_t S_t)R_t + A P_{t-1} A^T + q_t I \text{ and } G_t \text{ is the Kalman gain. Since for any other estimator of the response function, for instance } h^*_t = h_t(q^*T q), \\
E[(h_t - h^*_t)'(h_t - h^*_t)] \geq tr[P_t(q)], \text{ when the innovation variance is not known exactly, a good estimator of it improves channel response estimation.}
\]

### 2.1 Approximate MAP estimate

The marginal density

\[
p(q | r,e) = \hat{\phi} p(r_t | h_t, q, r,e) p(h_t | q, r,e) dh_t = p(q | r,e)
\]

leads to a MAP estimate of $q$ given $r,e$ as

\[
\hat{q}_{E_t} = \arg \max_q \log \hat{\psi}_q(S_t, \hat{h}_t, A P_{t-1} A^T + q I + S_t)^{-1} p(q | r,e).
\]

Estimates based on approximations to this criteria can be used to augment a Kalman recursion for improved estimation of the response $h_t$. Figure 3 presents estimates based on the MAP principle above for $q_{E_t}$ for the 8 phone array displayed relative to phone 4.

3. **RESULTS**

Figure 4 displays a channel response function and a histogram of $\frac{tr[S_t P_t S_t]}{tr[S]}$ which measures adversity of response estimation uncertainty relative to ambient noise variance. The probing interval is of duration 6.25 seconds at a range of approximately 500m. The channel uncertainty is two times as adverse to communications than the noise variance.

### REFERENCES


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