1. **INTRODUCTION**

Even with the use of high bypass ratio turbofan engines, jet noise continues to be an important contributor to the total aircraft noise on takeoff. In the case of military fighter aircraft engines, with negligible bypass ratio, jet noise is the dominant noise source at all flight conditions. Thus, jet noise is an important component of community noise in the vicinity of both commercial airports and military bases, and jet noise reduction is an ongoing problem for engine manufacturers. However, after nearly sixty years of research, the prediction of jet noise continues to be a challenging problem. This is especially true if rapid estimates, such as necessary for acoustics to become part of the engine design cycle, are needed. In most instances, such predictions are either based on a company’s proprietary experimental database or are based on a Reynolds-averaged Navier-Stokes (RANS) simulation for the jet’s mean flow, coupled with an acoustic analogy. Unfortunately, such predictions are often in error, particularly in the peak noise radiation direction. In the present paper the likely reasons for this difficulty are described.

The focus of the present paper is on noise predictions based on relatively simple models. However, there is no question that numerical simulations have made tremendous progress in recent years. Detailed calculations of the full time-dependent, three-dimensional near field, coupled with an extrapolation method to extend the solution semi-analytically to the far field, have shown great promise. For example, Bogey et al. (2003) performed a Large Eddy Simulation of a high subsonic jet. Their calculations also included the acoustic field as part of the simulation. Shur et al. (2005a, 2005b) also performed a Large Eddy Simulation in the near field and then extended the near field solution to the far field using a permeable surface implementation of the Ffowcs Williams-Hawkins (1969) acoustic analogy: see Brentner and Farassat (1998) for an excellent discussion of this technique. Shur et al. considered simple, single axisymmetric jets; jets operating off-design; coaxial jets with chevrons; and heated as well as unheated jets. It is an impressive study. However, these simulations take considerable time and computational resources. In the end, they represent a large numerical experiment and, by themselves, offer no insight into noise generation mechanisms. Without this insight, simpler models and schemes for noise reduction are less likely to be developed.

In the next section a brief review of previous theory and predictions is given. This is followed by a discussion of jet noise measurements, how well they support previous theory, and what help they can give in suggesting new directions. Finally, a simple model is proposed for the generation and radiation of noise by the large scale turbulent structures in the jet shear layer. It is argued that this mechanism, lacking in traditional acoustic analogy approaches, is dominant in the peak noise radiation direction at both subsonic and supersonic jet operating conditions.

2. **BACKGROUND**

The theory of aerodynamic noise was developed by Sir James Lighthill (1952, 1954). This was the first use of an acoustic analogy. In an acoustic analogy the equations of motion are rearranged into the form of an expression for propagation on the left hand side of the equation and the remaining terms are treated as equivalent sources. In Lighthill’s acoustic analogy the propagator is the wave equation in an undisturbed medium at rest. Lighthill identified the sources as having a quadrupole form – by analogy with classical acoustics. Using simple scaling arguments Lighthill deduced that jet noise power should scale with the eighth power of the jet exit velocity and the square of the jet exit area. Lighthill’s acoustic analogy was extended to turbulence convecting at high speed by Ffowcs Williams (1963). He argued that due to non-compactness effects, the power radiated by a jet should scale with three powers of jet exit velocity at sufficiently high Mach numbers. All these results were confirmed by the available experimental evidence. Features of the theory at this time included “convective amplification” and Doppler frequency shifts. These effects were argued to increase the noise radiation in the jet downstream direction with five inverse powers of the (modified) Doppler factor and shift the spectrum to higher frequencies. Again there was general agreement with experiments. However, measurements by Lush (1971) and others, showed that in the downstream direction the peak frequency actually decreased and that convective amplification over-predicted the sound pressure levels. It was argued by Lilley (1973) and Goldstein (1976) that these discrepancies could be explained if the propagation of the sound, once generated, through the non-uniform mean flow was included. This led to the development of Lilley’s equation which includes refraction effects. This is an acoustic analogy in which the propagator describes the propagation of sound through a non-uniform (in velocity and speed of sound) mean flow. In the special...
case of a parallel mean flow, which is a reasonable approximation to the actual slowly varying mean flow, the equivalent source terms are second-order in the fluctuations. General solutions to Lilley’s equation must be obtained numerically, but high and low frequency approximations are available: see, for example, Goldstein (1976), Mani (1978), and Balsa (1980). These solutions were coupled to a simple model of the jet flow by Balsa et al. (1978) and Morfey et al. (1978). This provided the first attempt to make noise predictions based on a flow solution, though the latter model used experimental data to define a master spectrum in the absence of mean flow/acoustic interaction effects.

In the theory and predictions outlined above, there are several common features. First, refraction effects are included in the propagation predictions. Second, a model is required for the statistical properties of the turbulence. In particular the two-point space-time correlation of the velocity fluctuations must be modeled. Ffowcs Williams (1963) assumed that the correlation took a Gaussian form in both space and time and this assumption was used for many years. It should be noted that Ffowcs Williams emphasized that this was a model, chosen for ease of analysis. It was not necessarily based on experimental observations. However, it is only with this model that five inverse powers of convective amplification are predicted. In fact, there are very few measurements of the two-point properties of the turbulence. The measurements by Davies et al. (1963) have often been used as guidance for the models – and they do not have a Gaussian form. Another feature of the analysis was the use of a moving reference frame to describe these turbulence properties. It was argued, correctly, that in a reference frame convecting with the turbulence, the sources could be treated as compact up to higher jet velocities. It also removes the apparently high frequency content of the fluctuations due to convection effects, which are unrelated to the frequency of the noise radiation. To a great extent, this represents the theoretical framework that is still used today for jet noise prediction. The primary advance has been the use of RANS solutions to describe the mean flow and the turbulence length and time scales: see Khavaran et al. (1994) for example.

In the present paper it is not possible to provide all the background analysis and only the appropriate references are provided. First, it is important to repeat that the five inverse powers of convective amplification are tied to the Gaussian model for the space-time correlation of the velocity fluctuations. Models based more closely on experimental measurements, such as those given by Harper-Bourne (2003), result in negligible convective amplification: see Morris and Boluriaan (2004) and Raizada and Morris (2006). In addition, Morris et al. (2002) showed that whether the turbulence statistics are described in a moving or stationary reference frame the same prediction for the radiated noise is obtained. Also, it should be noted that noise predictions at or near 90 degrees to the jet axis provide excellent agreement with experiment. An example is shown in Fig. 1. This prediction uses the method described by Raizada and Morris (2006). However, using the same acoustic analogy model, and including the mean flow/acoustic interaction effects as described by Lilley’s equation, predictions in the peak noise direction, close to the jet downstream axis, fail to match noise measurements.

3. EXPERIMENTAL OBSERVATIONS

In recent years there have been a series of experiments conducted in a high quality facility and new interpretations of experimental data that shed light on the prediction problems described at the end of the previous section. Viswanathan (2004) made measurements in single axisymmetric jets at a wide range of jet operating conditions. Prior to undertaking this study a careful examination of the quality of the experimental facility was conducted. Comparisons were also made with previous measurements in other anechoic jet facilities and problems with these prior measurements were identified. Details of the experimental facility and comparisons with other data are given by Viswanathan (2003). For example, previous measurements of jet noise from heated jets had suggested a change in the shape of the spectrum for heated, low speed jets. This data had been used, by Morfey et al. (1978) for example, to propose that a dipole-like noise source, generated by temperature fluctuations in the jet, was responsible for change in spectral shape. However, measurements in different diameter jets showed that the spectrum returned to the shape of the unheated jet spectrum as the diameter, and the Reynolds number, of the jet increased. This is a case where modelers were misled by measurements.
Before examining some of Viswanathan’s measurements another experimental observation is useful. Tam et al. (1996) examined and correlated a large database of noise measurements from NASA Langley Research Center and other facilities. They observed that the spectra at all angles to the jet and for a wide range of operating conditions could be collapsed using two spectral shapes. These were named the Fine Scale Similarity (FSS) and the Large Scale Similarity (LSS) spectra. The FSS spectrum is a broad, fairly flat spectrum and the LSS spectrum is much more peaked. The spectrum shapes are shown in Fig. 2. At large angles to the jet downstream axis the FSS spectrum alone fitted the data and at small angles to the jet downstream axis the LSS spectrum alone fitted the data. At intermediate angles the spectra could be fitted with a linear combination of the two spectral shapes. Tam et al. (1996) argued that their observations could be explained by the existence of two separate noise generation mechanisms. They argued that the LSS spectrum is generated by the large scale structures in the jet shear layer. Morris and Tam (1977) and Tam and Burton (1984) had shown how the large scale structures could generate noise when the structures travel supersonically with respect to the ambient speed of sound. They modeled the large scale structures as instability waves and explained the noise generation mechanism with a wavy wall analogy. This is discussed further below. The agreement between prediction and experiment in both the jet’s near and far fields gave no room for doubt that the large scale structures are the dominant noise source in convectively supersonic jets. What is surprising is that exactly the same spectrum shape is observed in the peak noise directions in convectively subsonic jets! Figure 3, from Viswanathan (2004) shows unheated jet spectra at jet exit Mach numbers ranging from 0.4 to 1.0. If the convection velocity of the large structures is assumed to be 70% of the jet exit velocity, then all these cases are convectively subsonic with respect to the ambient speed of sound. However, the LSS spectrum shape fits all the data very well. There is another striking feature in this figure. The spectral peak is independent of jet Mach number. If there were a Doppler frequency shift effect then the spectral peak should move to a higher frequency as the Mach number increases. This strongly suggests that there is no Doppler shift associated with the LSS spectrum shape. The traditional explanation for this observation is that there is a
would cause the higher frequency components of the spectrum to be reduced due to refraction. At the relatively low Mach numbers of the jets considered, these effects might be expected to be relatively small. Also, it is far from clear that the offsetting effects of several phenomena – Doppler frequency shift, convective amplification, and mean flow/acoustic interaction effects – would balance so beautifully that the spectral shape would remain unchanged with Mach number. A more rational explanation is that the noise radiation in the peak noise radiation direction is controlled by a different mechanism.

Additional experimental evidence for there being a different mechanism for noise radiation in the peak noise radiation direction has been given by Tam et al. (2007). For example, Fig. 4 shows the variation of overall sound pressure level (OASPL) with angle for a heated jet at different Mach numbers. The OASPL has been separated into two contributions by fitting the LSS and FSS spectra to the measured spectra. For angles close to the downstream jet axis (note that the angles are measured from the upstream axis in this figure) the measured spectrum can be fitted with the LSS spectrum alone. At larger angles from the jet downstream axis the measured spectra can be fitted by the FSS spectrum alone. At intermediate angles, both spectra are required to fit the experimental data. It is clear that the OASPL contributions from sources associated with the LSS and FSS spectra have a quite different behavior. The FSS source contributions increase only slightly as the angle to the downstream jet axis decreases whereas that from sources associated with the LSS have a much stronger variation. For example, there are changes of the order of 20 dB over 40 degrees. The other feature to note is that the peak OASPL occurs at approximately 150 degrees to the jet upstream axis for all but the highest Mach number case. In this highest speed case, the peak moves to a polar angle of 140 degrees.

The contribution to the OASPL from the sources associated with the FSS spectrum can be predicted with the traditional, acoustic analogy-based, methods. Clearly, there is little variation with angle, which is consistent with an absence of convective amplification. This is also reinforced by the observation that the variation with angle from the fine-scale sources is independent of jet Mach number. If convective amplification were present, even if the number of inverse powers of Doppler factor were less than five, the OASPL would vary more rapidly with angle as the jet exit velocity increased. For example, if three inverse powers of Doppler factor are assumed for convective amplification, as suggested by Harper-Bourne (2003), and the convection velocity is assumed to be 70% of the jet exit velocity, then there would be a 4 dB increase from 90 to 130 degrees, relative to the jet upstream axis, for the $M_j = 0.6$ case and an increase of 8 dB in the $M_j = 1.5$. If anything, the measurements show a smaller increase in the higher Mach number case.

It is clear that a different approach is required to make predictions of the noise from the LSS spectrum sources. A simple model for the source mechanism is given in the next section.

4. A MODEL FOR NOISE GENERATION BY LSS SPECTRUM SOURCES

It has long been realized that if the amplitude of a travelling wave generated in the vicinity of the jet is constant, and if its convection velocity is subsonic, relative to the ambient speed of sound, then the pressure fluctuations it generates decay exponentially with distance for the jet. This is well-explained using the wavy wall analogy. However, if the amplitude varies with axial distance then some sound radiation will be possible. Crow (1972), based on his observations of large scale structures in excited jets [Crow and Champagne (1971)], proposed a “line-antenna” model. This consists of a traveling wave with a Gaussian amplitude variation in the axial direction. This was also considered by Crighton (1975) and Ffowes Williams and Kempston (1978). Experiments in a low speed jet, motivated by the line antenna model, were conducted by Laufer and Yen (1983) who measured some of the growth and decay properties of the large scale turbulent structures. A formal matching of the instability waves in a two-dimensional shear layer with their acoustic radiation was described by Tam and Morris (1980) and this procedure was applied to axisymmetric jets by Morris and Tam (1977) and Tam and Burton (1984) – the latter paper providing a complete analysis of the asymptotic matching. In these papers, the large scale structures were modeled as instability waves supported by the mean flow. Their growth and decay were calculated from the linearized, inviscid equations of motion. These could be reduced to the so-called “Rayleigh equation” of hydrodynamic stability. Special care had to be taken when calculating the decaying stage of the wave, but comparisons with viscous calculations showed that the viscous results were a good approximation to the viscous solutions, except at very low Reynolds numbers. Tam and Morris (1980), in their calculations for the two-dimensional mixing layer, showed that the rate of growth and decay given by the linear instability analysis was not sufficiently rapid to lead to significant noise radiation. It should be noted that comparisons of linear instability wave models provide a good description of the growth phase of the large scale structures, based on comparisons with measurements using excitation: see Gaster et al. (1985) for example. However, it is unlikely that the rate of decay of the large scale structures is controlled by a linear analysis. This breakdown involves the nonlinear transfer of energy to smaller scale motions. The modeling and prediction of this breakdown process remains an unanswered problem.

However, it is still useful to reexamine the line antenna model. Assume that the pressure fluctuation generated on a
cylindrical surface of radius \( a \) in the near field of the jet can be written as\(^1\),

\[
p(a, z, t) = \text{Re}\{\hat{p}(z, t)\} = \text{Re}\{A(z)\exp[i(\alpha(z) - \omega t)]\}
\]

Then it is straightforward to show, using the method of stationary phase, that the far field pressure is given by,

\[
\hat{p}(R, \theta) = -\frac{i}{\pi R} \frac{P(a, \omega \cos \theta / c_o)}{H^{(1)}_o (\omega \sin \theta / c_o)} \exp(i\omega R / c_o)
\]

where \( P(a, s) \) is the Fourier transform of the pressure on the cylindrical surface with respect to the axial wavenumber \( s \) and \( c_o \) is the ambient speed of sound. Note that the axial wavenumber spectrum is to be evaluated at the wavenumber that gives a sonic velocity at a polar angle \( \theta \) relative to the jet downstream axis. \( H^{(1)}_o \) is the Hankel function of the first kind and order zero. In this simple model only an axisymmetric disturbance on the cylindrical surface has been considered. In addition, only a single frequency is examined. A more complete model would involve a random superposition of all frequencies and azimuthal mode numbers. For example, the surface pressure fluctuation could be written as,

\[
\hat{p}(a, \phi, z, t) = \int_{-\infty}^{\infty} \sum_{n=0} \bar{a}_n(\omega) \exp\left[i\left[\chi_n(z, \omega) + n\phi - \omega t\right]\right] d\omega
\]

where \( \chi_n(z, \omega) \) is the axial phase variation of the component with frequency \( \omega \) and azimuthal mode number \( n \). \( a_n(\omega) \) is a random amplitude of the \( n \)-th azimuthal mode with frequency \( \omega \). In addition, the surface surrounding the jet could be conical to allow for the spreading of the jet.

Returning to the simple, axisymmetric, single frequency case, assume that the wave has a constant phase velocity and a Gaussian amplitude envelope. That is,

\[
\hat{p}(a, z) = A \exp\left[-b(z - z_s)^2\right]\exp(i\alpha_o z)
\]

The corresponding solution for \( P(a, s) \) is,

\[
P(a, s) = A \left(\frac{\pi}{b}\right) \exp\left[-iz_s(\alpha_o - s)\right] \exp\left[-\left(\alpha_o - s\right)^2 / 4b\right]
\]

There are several things to notice. First, the peak in the amplitude of \( P(a, s) \) occurs at \( s = \alpha_o \). Thus, if \( \alpha_o > \omega / c_o \), the peak wavenumber components of the pressure signal will not radiate. Secondly, the width of the Gaussian envelope for the transform is controlled by the rate of amplitude change of the pressure with axial distance: the factor \( b \). Thus if the amplitude of the instability wave is changing very slowly, that is if \( b < 1 \), the transform’s amplitude will have a very narrow bandwidth. This corresponds to highly directional radiation and would give negligible radiation for subsonic convection velocities. This is the classical wavy wall problem – with constant amplitude. However, if there is a relatively rapid change in the amplitude of the wave packet, the bandwidth of the transform will be much greater and more radiating components will be generated.

Some example calculations have been performed to estimate the far field directivity associated with this model. The predictions, shown in Fig. 5, correspond to a Strouhal number of 0.2 and different convective Mach numbers. The polar angle is chosen to be relative to the jet upstream axis, to correspond to Fig. 4. It should be noted that the peak radiation direction is located at 150° for subsonic convection Mach numbers. This is due to the weighting by the Hankel function in the denominator in Eqn. (2) for the far field pressure. Also, as the convective Mach number becomes supersonic, the peak in the directivity moves to smaller polar angles. It should be noted that the Mach angle for \( M_c = 1.1 \) is equal to 165°, so the influence of the Hankel function weighting is still felt. However, at higher convective Mach numbers, the peak direction is controlled primarily by the phase velocity – it is 132° for \( M_c = 1.5 \). All of these features are consistent with the observations shown in Fig. 4.

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\(^1\) It should be noted that this wave packet representation has been used by several researchers. Most recently by Tam et al. (2007).
Calculations have also been performed for more general variations in the axial amplitude and phase, including more rapid amplitude decay than growth and a variable phase velocity. In these cases it is necessary to perform the Fourier transform numerically. However, the trends are as expected. Increasing the phase velocity moves more wavenumber components into the radiating region and increasing the amplitude variation broadens the directivity.

5. DISCUSSION

In the previous section a simple explanation of the observed jet noise radiation in the peak noise directions was proposed. It clearly provides a reasonable explanation of the measurements shown in Fig. 4. However, as noted in the previous section, a more general model is needed to include more frequencies and azimuthal mode numbers. In addition, there is really no prediction involved, as the amplitude and phase variations have been chosen arbitrarily. One could try to use noise measurements to work backwards to find the pressure variation that would give rise to those measurements. However, this is not as simple as it might appear. The problem is that only a limited range of wavenumbers will actually radiate to the far field: those for which \(|k| \leq \omega / a_s\). So the reconstructed pressure pattern in the near field would be based on a low-pass filtered version of the actual pressure signal. This is a common feature of far field phased array measurements when trying to reconstruct the source distribution. Another possibility would be to use a microphone array in the near field.

Even though such inverse methods would supply useful information, they would not provide a true predictive capability. There is a clear need for a model that can describe the axial phase and amplitude characteristics of the instability waves. A possibility would be to extend the linear stability wave model to a nonlinear one. A possible approach would be to use the Parabolized Stability Equations (PSE) [see Malik and Chang (2000) for an application to jet stability]. In the PSE multiple modes and their interactions can be included, though still within the framework of locally linear analysis. Some progress along these lines has been made by Cheung et al. (2007). However, they did not pursue the complete nonlinear capability of the PSE. The development of a predictive model is clearly a need for the future of jet noise prediction.

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