PSYCHOLOGY-BASED RHYTHMIC TRANSFORMATIONS

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1. INTRODUCTION

In most musical traditions rhythmic transformations including rephrasing, displacement, changes of division, metric modulation and segmentation are applied to rhythmic cells (Anku, 1997). Many examples may be drawn from music with developed percussion such as that in the Afro-Cuban tradition, the Indian tabla and Indonesian gamelan traditions, or jazz music. Rhythmic transformations have also been used to study the evolution of music. For example, Pérez-Fernández proposed a theory of how rhythms in some regions of Latin America may have originated from West-African rhythms, and consequently how one musical tradition may have evolved from another as a result of cultural interaction (Pérez-Fernández, 1986) .

Musical cyclic rhythms with a cycle length (timespan) of 8 or 16 pulses are called *binary*; those with 6 or 12 pulses are called *ternary*. The process of mapping a ternary rhythm of, say 12 pulses, to a rhythm of 16 pulses, such that musicologically salient properties are preserved is termed *binarization*. By analogy the converse process of mapping a binary rhythm to a ternary rhythm, proposed here, is referred to as *ternarization*. Pérez-Fernández's theory relies heavily on this binarization transformation. However, this theory has its opponents. Indeed several critiques of this theory have already been published. Gómez et al., (2007) introduced a class of purely mathematical rhythmic transformations, called snapping rules, to binarize and ternarize rhythms, which snap onsets of one representation to onsets of another representation on the basis of proximity rules. However, since the listener has the last word in judging music, the perceptual standpoint should not be overlooked.

Neil McLachlan in *A Spatial Theory of Rhythmic Resolution* (McLachlan, 2000) analyzed music from the African and Javanese musical traditions using Gestalt theory and mathematical group theory. Some of his constructions actually lead to our snapping rules for binarization. In this paper we generalize McLachlan's constructions to include ternarizations and other timespans such as 6 and 8. A second goal of this paper is to test the snapping rules derived from McLachlan's rhythmic constructions. Following the approach of Gómez et al. (2007), we compare our results to those of Pérez-Fernández. More specifically, we tested the validity of McLachlan's constructions as music-perception-based binarization and ternarization transformations.

2. RHYTHMIC TRANSFORMATIONS AND GESTALT PSYCHOLOGY

Prompted by the idea that onset proximity should be a criterion to maintain perceptual resemblance among rhythms, Gómez et al. defined the *nearest neighbour* rule (NN) and the *furthest neighbour* rule (FN). The nearest neighbour rule snaps an onset to the nearest neighbour, whereas the furthest neighbour rule snaps it to the furthest neighbour. They also introduced snapping rules based on direction such as the *clockwise neighbour* rule (CN), which moves an onset to the next neighbour in a clockwise direction; and *counter-clockwise neighbour* rule (CCN), which moves the onsets in a counter-clockwise direction. Onsets belonging to both pulses are snapped to themselves (i.e., they do not move). It may seem counter-intuitive to use the FN rule. However, in our study this rule was used to fully understand the snapping process, and because it actually gave good results in some cases. We note that snapping rules have been used previously in the study of rhythmic patterns in the context of automatic pattern generation and the formal definition of families of rhythms (Toussaint, 2002), (Demaine et al., 2005).

McLachlan used several Gestalt psychology principles to define rhythmic transformations (McLachlan, 2000). Of particular interest is his transformation from a 12-pulse rhythm to a 16-pulse rhythm. In the presence of a rhythm with timespan length divisible by 4, the superposition of two groups of 3 pulses each over two groups of 4 pulses each is, from the perceptual point of view, sufficient to generate a 12-pulse rhythm (refer to Figure 1). Following the Gestalt principle of simplest organization, this is the minimum information (or minimum resolution in the terminology of McLachlan), up to pulse 8, required to yield a 12-pulse rhythm.

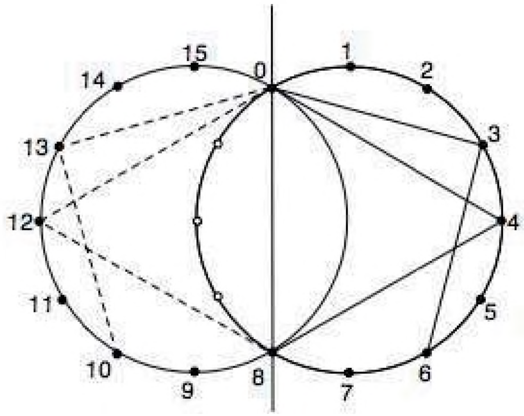

Fig. 1. McLachlan's transform ation for 12-pulse rhythms.

At this point, a mirror-reflection of the 12-pulse rhythm across the line joining pulses 0 and 8 is carried out. The final result is a 16-pulse rhythm where certain pulses are perceptually more salient than others; those are the pulses at positions 0, 3, 4, 6, 8, 10, 12 and 13 (see Figure 1). Mclachlan applies these ideas to the analysis of the clave son [x . . x . . x . . . x . x . . .], and reports that all the onsets of the clave son are on pulses defined by this construction. Pertinently enough, this construction implies a mapping rule for the problem of binarization. Indeed, snap the 12-pulse rhythm to the 16-pulse rhythm using the nearest neighbour rule, but restrict the snapping of the given rhythm to pulses numbered 0, 3, 4, 6, 8, 10, 12 and 13. For example, the binarization of the bembé bell pattern ubiquitous in West Africa (Toussaint, 2002) given by [x . x . x x . x . x . x] is the clave son [x . . x . . x . . . x . x . . .]

3. RESULTS and CONCLUSIONS

In the book of Pérez-Fernández the rhythms he gathered are classified into three groups: (1) ternary rhythms having 6-pulse timespans; (2) ternary rhythms having 12-pulse timespans; (3) ternary rhythms having 6-pulse timespans that are formed by variations of the molossus [x . x . x .], the first part of the clave son.

The binarization experiments with 6-pulse ternary rhythms discussed by Pérez-Fernández, yielded no matches. Some instances produced mappings that are somewhat surprising. For instance, a rhythm with a clear perceptual structure such as [x x x x x x] is binarized to [x . x x x x x .], instead of something like [x x x x x x . .]. Also [x x x . x x] is binarized to the Cuban cinquillo [x . x x . x x .]. On the other hand, when the original rhythm has many onsets in common with the salient positions in the construction of McLachlan, the binarization works reasonably well. For example, the binarization of the choriamb, takes [x . x x x .] to [x . . x x x . .]. For 12-pulse rhythms there was only one match taking the ternary clave son [x . x . x . . x . x . .] to the binary clave son. The bembé, which contains the 6/8 clave son, is also binarized to the clave son.

Since there is as yet no theory for ternarizations, we used his binarization examples as the set of input rhythms for ternarization. No matches to Pérez-Fernández's rhythms were found with any of the rhythms. The ternarized rhythms seem to have little perceptual resemblance with their binary counterparts. In particular, the binarized bembé does not ternarize to the original bembé.

In conclusion, the rules based on Gestalt psychology devised by McLachlan appear to have limited applicability. They work reasonably well for the binarization of 12-pulse rhythms and ternarization of 16-pulse rhythms, but not for other timespans, particularly those of small duration. Furthermore, breaking expectancy exactly at the middle of the timespan is not suitable for rhythms that do not have this kind of perceptual structure. In this study we considered only the rhythms addressed in Pérez-Fernández's book. We feel that they are too few for us to draw strong general conclusions. Therefore, it would be desirable to repeat this study on a broader range of rhythms, taken from world music, where a greater variety of metric patterns would permit an in-depth study of perceptual-based rhythmic transformations.

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