

# MODELING THE VIBROACOUSTIC RESPONSE OF MULTI-MATERIALS COMPLEX STRUCTURES UNDER MECHANICAL EXCITATION

Dilal RHAZI<sup>1</sup>, Nouredine ATALLA<sup>2</sup>

<sup>1,2</sup>GAUS, Dept. of mechanical engineering, University of Sherbrooke, Qc, Canada

## 1. INTRODUCTION

The prediction of the acoustic and vibration behavior of a multi-layer structure, made up of parallel and homogeneous layers is of interest to several industries. A typical configuration consists of a master structure with an attached Noise Control Treatment (NCT) in both single wall and double wall configurations. The Transfer matrix methods (TMM) is extensively used and well validated for solving the above problem (Allard [1], ATALLA [2] and MUNJAL [3, 4]). However, it is mainly limited to acoustic excitation (plane wave or diffuse field). Few works have been published on the use and validity of the method for a structure-borne excitation. For instance, Villot and all [5] rapidly hinted to the use of the TMM to solve the response of a multilayer with a mechanical excitation, including correction for the size effect using spatial windowing; but didn't present example or validation results.

Therefore, a rigorous work is still lacking for the mechanical excitations (structure-borne). The presented work concentrates on the latter. It presents the study and validation of three different approaches to model the vibration and acoustic response of a mechanically excited structure with added NCT. The first approach is based on the propagation of plane waves in the main structure and the layers of the NCT. The second uses SEA (Statistical Energy Analysis) for the main structure and calculates an equivalent damping to account for the NCT. The last approach uses the modal technique by calculating the equivalent impedance for the added treatment. In the three approaches, the transfer matrix method is used to model various multilayered acoustic control treatments. Applying these approaches to some aircraft structures confirm their relevance in relation to more exact and costly methods, such as the finite elements method.

## 2. THEORY

The Multilayer separates two semi-infinite fluids. Moreover the mechanical excitation and the response of the structure are assumed harmonic. In the first approach (referred to by the wave approach), the studied structures are assumed of infinite extent and excited by a point load  $f(x, y)$ . In wavenumber space  $(k_x, k_y)$ , the field

$f(x, y)$  can be considered to be constructed from an infinite number of plane waves. For each wavenumber, Transfer Matrix Method is used to compute the input and output (transfer) indicators (velocities, impedance, pressure, etc.). For example, the quadratic velocity of the main structure takes the form:

$$\langle \bar{v}^2 \rangle = \frac{1}{8\pi^2 S} \int_0^{2\pi+\infty} \int_0^{\infty} \left| [V(k_x, k_y)]_{z=0} \right|^2 k_r dk_r d\varphi$$

$$\text{Where: } [V(k_x, k_y)]_{z=0} = \frac{1}{Z_{S,TMM} + Z_{B,\infty}}$$

$Z_{S,TMM}$  is the impedance of the multilayer, seen from the excitation side. It is calculated using the TMM.

$$Z_{B,\infty} = \frac{k_0 Z_0}{\sqrt{k_0^2 - (k_x^2 + k_y^2)}} \text{ is the radiation impedance, seen}$$

from the emission side,  $k_0$  the acoustic wavenumber,  $Z_0$  the characteristic impedance in the emission domain, and  $S$  is the surface of the multilayer.

Note that the panel's size is accounted for in the calculation of the acoustic indicators using the Finite Transfer Matrix Method (FTMM). The application of this method for calculation of the radiated power is shown by the following equation:

$$\pi_{rad} = \frac{1}{8\pi^2} \left[ \int_0^{2\pi} \int_0^k \frac{Z_0 \sigma_{finie}(k_r, \varphi)}{|Z_{B,\infty} + Z_{S,TMM}|^2} k_r dk_r d\varphi \right]$$

$$\text{Where: } \sigma_{finie} = \frac{\Re(Z_R)}{Z_0 S}$$

And:

$$Z_R = i\omega\rho_0 \iint_{s \ s} e^{-i(k_x x_0 + k_y y_0)} G(M, M_0) e^{i(k_x x + k_y y)} dS(M_0) dS(M)$$

This term represents a geometrical correction to account for the finite size effect. And it depends only on the geometry

of the panel.  $G(M, M_0) = \frac{\rho^{-ik_0 R}}{2\pi R}$  is the half space

Green's function. The principle of the approach is to replace the radiation efficiency in the receiving medium by the radiation efficiency of an equivalent baffled window. Let us note that the approach is valid strictly for planar structures. Figure 2 shows the relevance of the technique for an aluminum plate of thickness 0.003 (m).

In the second approach, based on SEA for the main structure, a light coupling is assumed between the structure and the sound package. The effect of the sound package is accounted for through an equivalent damping  $\eta_{eq}$ , the latter is calculated for each structural wavenumber, from the TMM applied to the NCT in a piston motion.

The same methodology is used for the modal approach (third approach). The first layer is modeled by a modal technique. Next, the transfer matrix methods is used to evaluate for each mode  $(m, n)$  the expression of the equivalent impedance of the treatment  $Z_{eq, mn}$ . The total impedance:  $Z_{eqT} = Z_{mn}(Bar\ plate) + Z_{eq, mn}$ .

### 3. RESULTS

In this section the comparison between the three presented approaches is illustrate by the example of a double panel made up from a Plate-Fiber-Plate. The aluminum plates have the same thickness:  $h=0.003$  (m) and the fiber of thickness  $H=0.01$  (m) is modeled as a limp porous layer. The force has unit magnitude, and the results are plotted in third of octaves. In order to compare the three methods, a comparison to the finite element method (FEM) is presented. The FE prediction is carried out with the Novafem software, developed at GAUS. The plates were modeled using xyz cquad4 elements while the fiber was modeled using yx bricks equivalent fluid elements (limp porous) elements. The plates were assumed baffled for acoustic radiation. Due to the cost of the FE calculation, the calculations were done for a fixed force position while the presented approaches assume a rain on the roof type of excitation. A comparison of the quadratic velocity computed using the three presented approaches and the FEM is illustrated in figure 1. Good agreement is observed between the FTMM, modal and FE calculations. The latter depicts modal fluctuations at lower frequencies (recall that data are presented in 1/3 octave bands). The SEA approach overestimates the quadratic velocity. This is related to the assumption of slight coupling that only accounts for equivalent damping while neglecting the effects of stiffness and mass of the treatment.

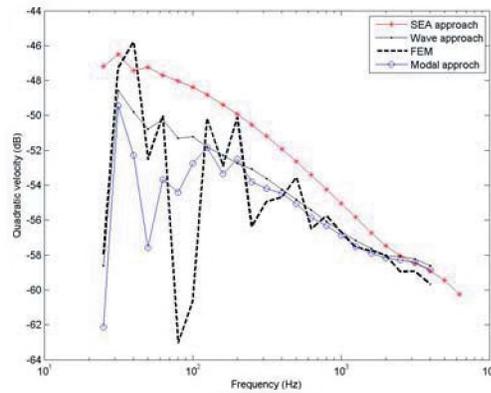


Fig.1. Quadratic velocity of a Plate-Fiber-Plate system excited by a random force.

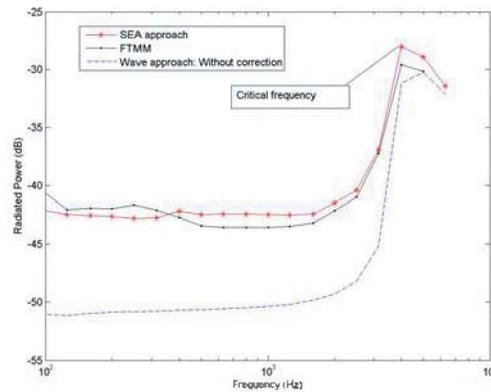


Fig.2. Radiated power of a single plate excited by a random force.

### 4. CONCLUSION

Three methods are presented and compared for the quick estimation of the structure-borne response of complex flat structures with added sound packages. They represent an attractive alternative to finite element method for quick assessment of Sound package effects. The three methods are shown to work correctly within their assumptions. A combination of the methods can be used to eliminate the assumption of low coupling classically assumed in SEA calculations.

### REFERENCES

- [1] Allard, J.F. (1993) Propagation of sound in porous media: modeling sound absorbing materials, Elsevier, New York.
- [2] Atalla, N. (2004). An overview of the numerical modeling of poroelastic materials. Congrès SAPEM, France.
- [3] Munjal, M, L. (1993) Response of a multi-layer infinite plate to an oblique plane wave by means of transfer matrices, Journal sound and vibration 162 (2), 333 p.
- [4] Munjal, M, L., Sastry, J, S. (1995) A transfer matrix approach for evaluation of the response of a multi-layer infinite plate to a two-dimensional pressure excitation, Journal sound and vibration 182 (1), 109 p.
- [5] Villot, M., Guigou, C., Gagliardini L. (2001) Predicting the acoustical radiation of finite size multi-layered structures by applying spatial windowing on infinite structures. Journal of Sound and Vibration 245(3), 433-455.