A NEW METHOD OF ACOUSTICAL REMOTE EVALUATION OF DEFECTIVE STRUCTURES AND MATERIAL CHARACTERIZATION

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1. INTRODUCTION

This study aims at showing the feasibility of a remote monitoring of structures for a progressive damage assessment as well as material characterization using a simple and inexpensive experimental setup. The method is based on a remote acoustic excitation of transverse vibrations on a membrane using an ordinary broadband low frequency loudspeaker, and the measurement of the response using a Laser Doppler Vibrometer (LDV). Small rectangular strips of aluminium foil (6.35 μm thick), Low Density Polyethylene (LDPE 25μm thick), Paperboard (PPR 100 μm thick) or a combination of these materials were considered. The samples were tested using the implemented Low Frequency Vibration-Based Technique (LFVBT), showing that remote damage monitoring and material characterization are feasible with this method. Theoretical modeling is also developed to correlate the experimental results obtained. This yields a new method for Non Destructive Testing (NDT) of sheet-like materials.

2. METHOD

2.1 Governing equation and steady-state vibration

The theoretical analysis in this study uses the theory of vibrating membrane with account of intrinsic elasticity, governed by the equation [1]:

$$\frac{\partial^2 \xi}{\partial t^2} - c^2\left(\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2}\right) + d^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \xi = \frac{p(x,y,t)}{\rho h} \quad (1)$$

Here ξ- is the displacement normal to the plane (xy) coincident with the equilibrium position of membrane, c - is the velocity of propagation of bending waves at zero intrinsic elasticity defined below, p - is external pressure on the surface of the membrane, ρ, h - are density and thickness of the material, and:

$$c = \sqrt{T/(\rho h)}\,; \quad d^2 = \frac{Eh^2}{12\rho(1-\nu^2)} \quad (2)$$

Where E, T and ν are the elastic modulus, tensile force per unit length of the edge, and the poisson's ratio.

The frequency equation with fixed-fixed boundary conditions was derived [2] to obtain the vibration frequency of each bending mode as follows:

$$\omega_{mn} = c\sqrt{\left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{b}\right)^2}\left\{1 + \frac{d^2}{2c^2}\left[\left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{b}\right)^2\right]\right\} \quad (3)$$

a, b are the dimensions of the membrane, m and n are the mode numbers.

2.2 Local density variation: addition of mass

Let the mass m' be attached to the sheet and assume its size to be small in comparison with the wavelength of bending wave.
This mass vibrates together with the membrane and yields the following equation of natural frequencies [3]:

$$\omega_{mn} = \Omega_{mn}\sqrt{\frac{1 + \frac{d^2}{c^4}\Omega_{mn}^2}{1 + 2\beta_q \frac{m'}{M}\cos^2\left(\pi m \frac{x_0}{a}\right)\cdot \sin^2\left(\pi n \frac{y_0}{b}\right)}} \quad (4)$$

where $M = \rho\, hab$- is the mass of the membrane, $\beta_0 = 1$, $\beta_q = 2$ ($q > 1$), and Ω_{mn} is the natural frequency of the unloaded membrane.
One can see that the local load shifts down the natural frequency. The shift is determined by the ratio of loading mass and the mass of membrane m'/M . This shift can be detected only if the special mode is excited which has the loop neighboring the point x0, y0 of the location of the loading mass. Evidently, the weight placed in the node cannot shift the frequency of the corresponding mode.

2.3 Young's modulus from dynamic measurement

From equation (2) relating the velocity of bending wave and the tensile load T, one can easily derive the following:

$$c = \sqrt{T/(\rho h)} = \sqrt{E\varepsilon/\rho} \quad (5)$$

where ε is the longitudinal strain.

Inserting equation (5) into equation (3) with no account of intrinsic elasticity, and solving for normal modes along the length of the specimen yield [2]:

$$f_{0n}^2 = \frac{E \cdot n^2}{4 \cdot \rho \cdot b^2} \cdot \varepsilon \quad (6)$$

where f_{0n} is the resonance frequency of mode $0n$ in Hz. E, n and ρ being all constants for a given measurement, the square resonance frequency is linearly proportional to the strain. As a consequence, dynamic Young's modulus is extracted from the slope of the curve.

Equation (6) offers an idea of how one can estimate Young's modulus of a material from a dynamic measurement using as inputs the longitudinal strain and the measurement of the corresponding resonance frequency in flexural mode.

3. EXPERIMENT AND RESULTS

The function generator provides an input voltage (peak to peak) of a sine signal to the loudspeaker. The acoustic field excited by the loudspeaker vibrates the sample. Laser detection of the surface vibrational response of the sample is accomplished with the laser vibrometer. The vibrometer used in this study makes high-fidelity and absolute measurements of surface displacement over a bandwidth of DC to 50 kHz. The measured response is monitored by the oscilloscope; this allows the detection of the maximum surface displacement, corresponding to a normal mode, which is related to the mechanical properties of the material, or carries information on any structural change in the sample under investigation. The setup is shown in figure 1.

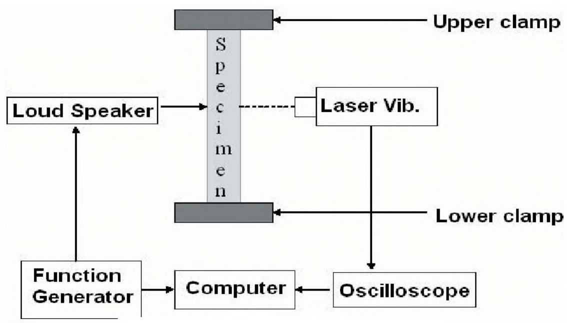

Figure 1: Experimental setup

Small strips are successively added in the middle of the sample and the fundamental frequency change is monitored. The result is shown in figure 2, together with a plot of equation (4), in terms of first mode relative frequency shift with respect to percentage of added mass. A good agreement is observed between experimental and analytical results.

For a varying strain (or extension) within elastic region of the material, equation (6) was plotted using experimental data for each sample. The slope of the line obtained, according to equation (6), allows the extraction of Young's modulus. The result is shown on figure 3 for paperboard, for different specimen lengths. Extended results for Al foil, LDPE, and laminates are published in [4].

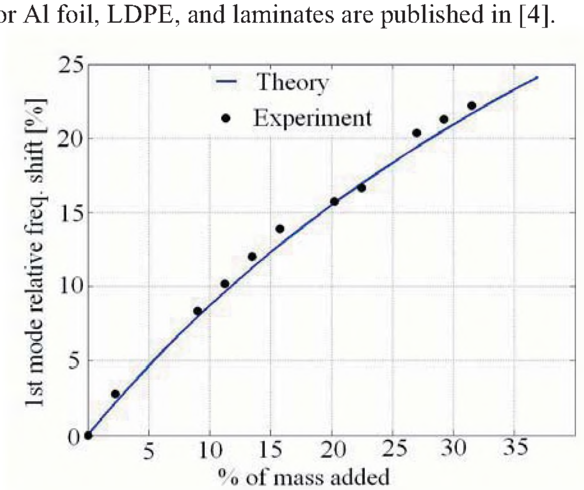

Figure 2: 1st mode relative frequency shift

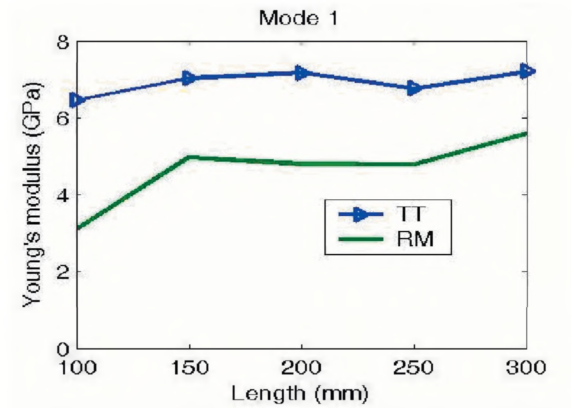

Figure 3: Young's modulus from static & dynamic tests.
TT = tensile test, RM = resonance method.

4. CONCLUSION AND DISCUSSION

The theory developed above offers the ability to detect an inhomogeneity in form of added mass using a frequency shift measurement. In other words, we use *acoustic weighing* of the specimen. We have also demonstrated the feasibility of Low Frequency Vibration-Based Technique for dynamic characterization of sheet materials. As they are both frequency measurements, one can expect that the sensitivity of this technique will be acceptable for non-destructive testing of sheet materials.

The observation that changes in structural properties cause changes in vibration frequencies is the impetus for using modal methods for health monitoring and material characterization. However, the method presented in this paper is certainly liable to some improvements. Thus, the combination of constitutive models and optimization algorithms [5] in a future investigation may lead to improvement of the suitability of the presented method for nondestructive characterization of vibrating sheets.

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