FREQUENCY ANALYSIS OF THE ACOUSTIC PRESSURE OF NONLINEAR STANDING WAVES

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1. INTRODUCTION

Analytical, numerical and experimental analysis of linear and nonlinear standing acoustic waves in time and space domains have been extensively investigated in the recent years[1-3]. When the amplitude of acoustic standing wave is infinitesimal, the acoustic wave can be described by linear laws. However, when the acoustic wave is driven into large amplitude oscillations, the equations of motion are nonlinear and the nonlinear effects could distort originally harmonic waves, shift the resonance frequency, and transform acoustic energy into higher harmonic components[4]. In this paper, we have examined the acoustic pressure inside a nonlinear standing wave resonator in the frequency domain, and the effect of nonlinearity on the frequency spectrum of the pressure signal through numerical and experimental investigations.

2. METHOD

The description of nonlinear acoustic waves in a viscous, heat-conducting fluid in 1-D is obtained using the following equation[5],

$$\rho_0 u_{tt} = \gamma p_0 \frac{1}{(1+u_x)^{\gamma+1}} u_{xx} + b u_{txx}, \qquad (1)$$

$$b = \kappa\left(\frac{1}{c_V} - \frac{1}{c_p}\right) + \frac{4}{3}\mu + \mu_B$$

where u is the particle displacement, ρ_0 and p_0 are the static density and pressure, respectively, μ and μ_B are the shear and bulk viscosities, κ is the coefficient of thermal conduction and $\gamma = c_p/c_V$ is the ratio of specific heats at constant pressure and constant volume. This nonlinear differential equation is solved numerically using an effective finite difference method (FDM) without truncation. The following initial and boundary conditions are applicable,

$$u(0,t) = u_0 \sin(\omega t) \;\;,\;\; u(L,t) = 0,$$
$$u(x,0) = 0 \;\;,\;\; u_t(x,0) = 0. \qquad (2)$$

The acoustic pressure p', can be evaluated from u using

$$p' = \frac{p_0}{\left(1+u_x\right)^{\gamma}} - p_0.$$

In the experimental part of this study, we have developed a setup to measure high-amplitude pressure inside the standing wave tube. The setup consists of a rigid-walled tube of adjustable length, a driving system to excite the standing wave inside the tube, and data acquisition system (see Fig. 1). The acoustic pressure is measured using a quarter-inch condenser microphone cartridge Model 377A10 PCB Piezotronics. The pressure signal from the microphone was acquired via a 16-channel data acquisition card (PCI-6036E, National Instruments) using the LabView data acquisition software. The measurements were made at five different locations along the channel center axis, at different excitation voltages. The numerical model requires maximum diaphragm displacement (u_0) for the boundary condition (see Eq. 2). A Bruel & Kjær laser vibrometer is used to measure this parameter for different loudspeaker driver intensities.

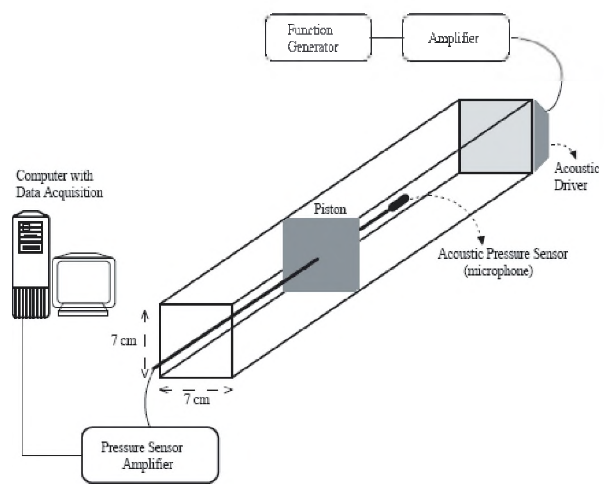

Figure 1: Schematic of the experimental setup and instrumentation.

3. NUMERICAL RESULTS

The frequency spectrum for the numerical and experimental pressure at different axial distances for u_0=175 μm and resonance frequency of f=1024 Hz are shown in Fig. 2. It can be seen that there is a good agreement between the numerical and experimental data in frequency domain. The existence of the higher harmonics in the frequency spectrum shows that in nonlinear acoustic resonators, acoustic energy is transformed into higher harmonic components. The plot shows that the fundamental mode is dominant at all spatial locations except at x = 8.5 cm, i.e. the middle of the channel (the location of the theoretical pressure node), where the second harmonic is dominant. The reason for this trend is that the pressure node is not fixed in time and space. That is, during a wave period, the pressure node oscillates about the theoretical pressure node. The frequency of oscillation is twice the resonance frequency. It is because of this oscillation the large amplitude second harmonic mode of pressure manifested at the location of theoretical pressure node in Fig. 2. The plot also shows that except at x = 8.5 cm, the magnitude of second harmonic from experimental results is higher than the numerical results. Vanhille and Campos-Pozuelo[1] studied the pressure amplitude at different harmonics at the center of the reflector. They also observed that the experimentally obtained pressure amplitude at second harmonic is overall higher than the numerical one. Using the verified model presented here, we can analyze the frequency spectrum of the pressure and particle velocity signals at different frequencies and excitation amplitudes from linear to finite-amplitude nonlinear cases.

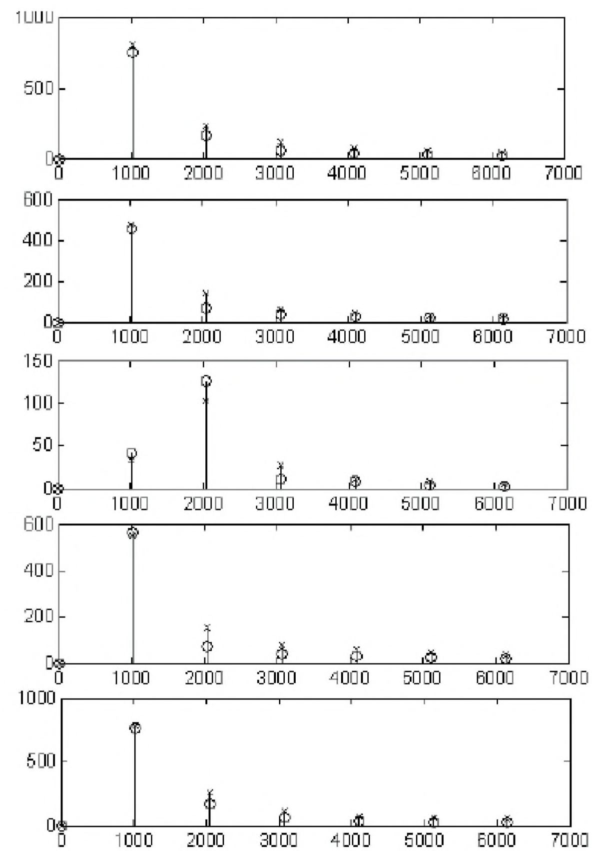

Figure 2: Frequency spectrum of the acoustic pressure for u_0 = 175 μm at (a) x = 17cm, (b) x = 13 cm, (c) x = 8.5 cm, (d) x = 5 cm and (e) x = 1 cm. ○, numerical; ×, experimental. Vertical axis, pressure in Pa; Horizontal axis, frequency in Hz. x is measured from the driver end of the resonator.

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