MONTE CARLO SIMULATION FOR CALCULATION OF UNCERTAINTY IN RECIPROCITY CALIBRATION OF MICROPHONES

Peter Hanes
Institute for National Measurement Standards, National Research Council Canada,
1200 Montreal Rd., Ottawa, Ontario, Canada, K1A 0R6 • peter.hanes@nrc-cnrc.gc.ca

1. INTRODUCTION

1.1 Reciprocity technique

Reciprocity calibration of the sensitivity level of a Laboratory Standard microphone is the means by which national measurement institutes provide traceability to the SI for acoustical measurements. The pressure reciprocity technique[1] requires the measurement of the electrical transfer impedance of coupled pairs from a set of three microphones and the calculation of the acoustic transfer impedance of the microphone pairs. The acoustic transfer impedance depends on the dimensions of the microphones and coupler, the acoustical properties of the microphones, the properties of the air in the coupler and the prevailing environmental conditions. Even with the well-defined geometry of LS1P and LS2P (Laboratory Standard) microphones[2], the model used for the calculation of acoustic transfer impedance is complicated.

1.2 Determination of measurement uncertainty

Calculation of the measurement uncertainty by determination of sensitivity coefficients and standard uncertainties for each input quantity[3] for the reciprocity technique is a time-consuming task. About 50 quantities that contribute to the uncertainty have been identified, many of which appear several times in the model. The sensitivity coefficients and standard uncertainties vary with frequency.

Monte Carlo simulation is a numerical technique that can be used to propagate the distributions of the input quantities using the calculation model in order to estimate the distribution of the outputs[4]. It can provide a standard uncertainty and a coverage interval for a desired level of confidence (typically 95 %) for the pressure sensitivity levels, without intermediate determination of sensitivity coefficients.

2. IMPLEMENTATION

Calculation software has been developed at NRC-INMS to compute the pressure sensitivity levels from the input data using the model of acoustic transfer impedance specified in IEC 61094-2:1992[1]. Corrections for the effect of radial wave motion in a plane-wave coupler[5] were also programmed. The software was validated by comparing its output against that of commercially-available software[6] and (at two frequencies only) a Microsoft Excel spreadsheet.

The NRC-INMS software was further developed to allow the calculation of the associated measurement uncertainty by Monte Carlo simulation.

The number of Monte Carlo trials to be performed in the simulation is determined by the requirements for the resolution of the uncertainties (here, two significant figures). In this exercise, standard uncertainties were determined from the results of 10^5 trials.

The effect on the pressure sensitivity level of the distributions of individual input quantities to the model can be found by fixing all other input quantities at their best estimates. The effects of all input quantities were estimated for both LS1P and LS2P microphones at selected frequencies.

3. RESULTS

Typical pressure sensitivity levels for one LS1P microphone from reciprocity measurements are shown in Fig. 1.

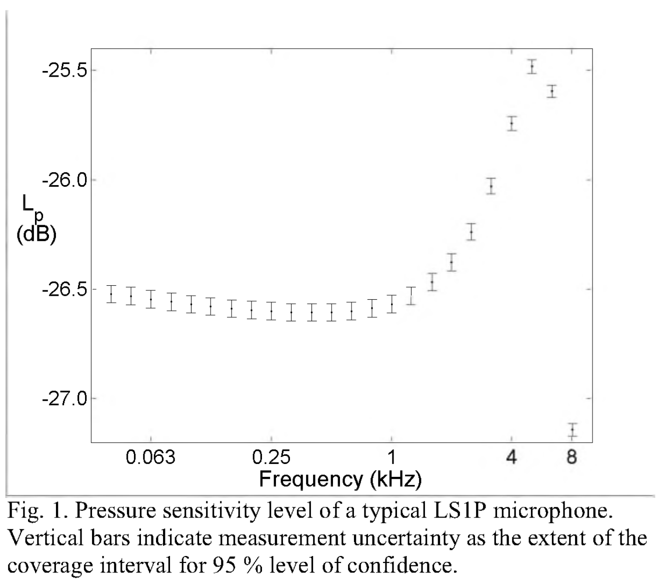

Fig. 1. Pressure sensitivity level of a typical LS1P microphone. Vertical bars indicate measurement uncertainty as the extent of the coverage interval for 95 % level of confidence.

A typical approximation to the probability density function for sensitivity level at one frequency is shown in Fig. 2.

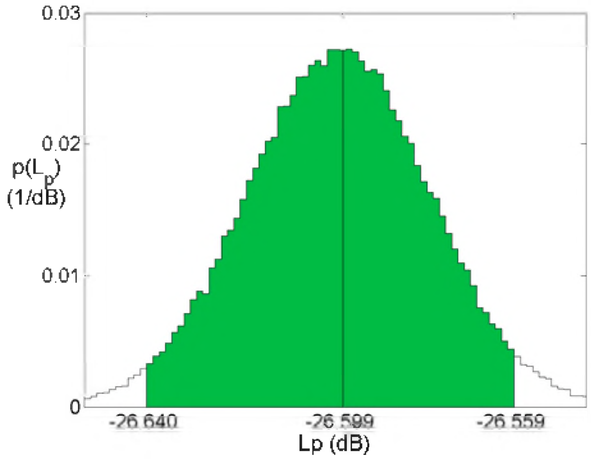

Fig. 2. Approximation to the probability distribution for the pressure sensitivity level of an LS1P microphone at 0.25 kHz, calculated by Monte Carlo simulation (10^5 events). The abscissa labels indicate the expected value and the extent of the coverage interval for 95 % level of confidence (shaded area).

The standard uncertainties were computed for frequencies at one-third-octave intervals from 40 Hz to 8 kHz (LS1P) or 20 kHz (LS2P); values for selected frequencies are listed in Table 1.

Table 1. Standard uncertainty in pressure sensitivity level computed by Monte Carlo simulation at selected frequencies for LS1P and LS2P microphones.

Frequency (kHz)	$u(L_p)$ (dB) for LS1P	$u(L_p)$ (dB) for LS2P
0.063	0.020	0.013
0.25	0.021	0.013
1	0.020	0.013
4	0.017	0.012
8	0.014	0.011
16	...	0.017
20	...	0.029

4. DISCUSSION

The standard uncertainties in Table 1 are in the expected ranges for the method and equipment used. The greatest contribution to the standard uncertainty at low frequencies is the uncertainty of the equivalent volume of the diaphragm when determining the acoustic impedance of the microphone. This contribution reduces as frequency increases. This pattern of frequency dependence confirms the results of a previous uncertainty determination based on use of partial derivatives of the model[7].

As the frequency approaches the resonance frequency of the diaphragm (about 8 kHz for LS1P or 23 kHz for LS2P) the greatest contributions arise from the unpredictability of radial wave motion and of viscous losses in the coupler.

Other significant uncertainty contributions are due to uncertainty in the sensitivity to static pressure of LS1P microphones, uncertainty in the reference value for speed of sound in dry air, and uncertainty in measurement of voltage ratio for electrical transfer impedance. The contributions from input quantities such as lengths and diameters of coupler and microphones are tightly controlled through precise dimensional measurements.

Evaluation of individual components enables future research and development work to focus on potential areas for improvement in uncertainty. Definition of the distributions of the input quantities and the implementation of the Monte Carlo simulation are subject to continuous improvement.

5. CONCLUSIONS

Probability distributions for pressure reciprocity levels of Laboratory Standard microphones can be determined using Monte Carlo simulation. The technique shows potential for use in determining measurement uncertainties for ‘primary’ acoustical standards.

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ACKNOWLEDGEMENTS

Won-Suk Ohm programmed the first version of the code for the calculation of the radial wave motion correction.