EFFECT OF MEAN FLOW ON THE ACOUSTIC TRAPPED MODES OF A CAVITY-DUCT SYSTEM

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1. INTRODUCTION

Flow-induced acoustic resonance in cavities, which is known to occur in numerous engineering applications, can cause hazardous noise levels, and in some cases catastrophic failure due to acoustic fatigue of the cavity structure [1]. Having the cavity attached to a confined domain reduces the acoustic radiation, resulting in "*trapped acoustic modes*" which are highly susceptible to self-sustaining acoustic resonance, even at moderate subsonic Mach number flow. In such configuration, the interaction between the acoustic field and the flow vorticity is the main source of acoustic power production.

In many previous studies, the acoustic field used in estimating the acoustic power was calculated at zero flow velocity, which ignores the effect of the mean flow on the resonant acoustic field. In this paper, the effect of mean flow on the embedded trapped acoustic mode of a two-dimensional cavity mounted at the middle of a rectangular duct is investigated numerically as a step toward improving the prediction of acoustic power.

The cavity-duct geometry is shown schematically in Fig. 1. Both cavity length (l) and depth (d) are taken to be 1/6 H, where H is the height of the duct. The duct length is slightly longer than 6 times the height to minimize the effect of the duct length and the ends boundary conditions on the shape of the acoustic mode. For all simulations, the direction of air flow is from the left to the right.

2. PROBLEM FORMULATION

2.1 Governing equations

The numerical method is based on solving the Acoustic Perturbation Equations (APE) developed by Ewert & Schroder [2]. The Acoustic Perturbation Equations account for the convection and refraction of the acoustic perturbation wave by the mean flow. However, it doesn't support the vorticity wave. This prevents the excitation of the hydrodynamic instability in the cavity free shear layer. As the coupling between the mean flow and the acoustic perturbation is outside the scope of this paper, aerodynamic sources are not considered in the solution. The APE system for multidimensional domain without source terms can be written as follow:

$$\frac{\partial p'}{\partial t} + \bar{c}^2 \nabla \cdot \left(\bar{\rho} \boldsymbol{u}^a + \bar{\boldsymbol{u}} \frac{p'}{\bar{c}^2} \right) = 0 \qquad (1)$$

$$\frac{\partial \boldsymbol{u}^a}{\partial t} + \nabla \left(\bar{\boldsymbol{u}} \cdot \boldsymbol{u}^a \right) + \nabla \left(\frac{p'}{\bar{\rho}} \right) = 0 \qquad (2)$$

Where p' is the acoustic pressure perturbation, u^a is the acoustic particle velocity vector, $\bar{u}$ is the mean velocity, ρ is the mean density and c is the mean acoustic speed, which is set to 340 m/s. The mean flow quantities used in the solution are calculated using the Reynolds Averaged Navier-Stockes equations Solver.

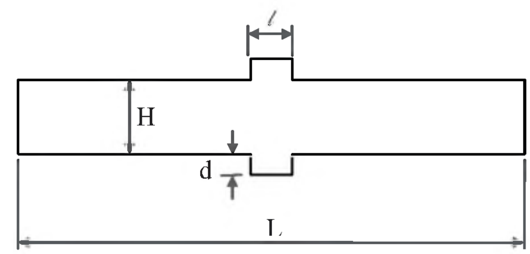

Fig. 1. Cavity-Duct Geometry

2 .2 Numerical algorithm

To solve the 2-D governing equations, a CAA code is developed. The spatial derivatives are calculated using the optimized prefactored compact scheme [3]. The time derivatives are calculated using 5/6 stage low dissipation and dispersion Runge-Kutta scheme in the 2N-Storage form [4]. A 6th order filter is also applied at each stage of the Runge-Kutta scheme to eliminate spurious short-wavelength disturbances that the numerical grid can't resolve. Regarding the boundary conditions, at the walls, the normal velocity is set to zero. At the duct ends, the acoustic pressure is set to zero which simulates a complete reflection of the acoustic wave. This condition is chosen because the main duct is sufficiently long to consider the actual radiation of the trapped modes to be negligible.

3. RESULTS AND DISCUSSION

In this part, the first trapped mode in the transverse direction to the flow, which is equivalent to the empty duct first transverse mode, is studied. The resonance mode is simulated at zero flow and at Mach numbers of 0.1, 0.2 and 0.3 based on the average velocity at the inlet of the duct. For each case, the resonance frequency is determined from the domain response to a narrow band external perturbation. The external perturbation is introduced to the domain as a velocity fluctuation at the cavity top and bottom floors in the normal direction to the wall. The top and bottom floors oscillate out of phase to excite the first transverse mode. Figure 2 shows the change of the resonance frequency with the Mach number. The first transverse mode resonance frequency experiences consistent drop as the Mach number increases. The rate of frequency decrease appears to be proportional to the square of the Mach number. Similar behavior was observed during a current experimental study focusing on aerodynamic excitation of transverse acoustic modes of axi-symmetric cavity-duct system.

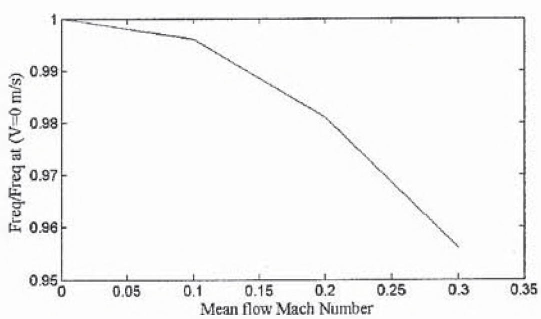

Fig. 2. Resonance frequency change with the Mach number

To obtain the mode shape, the domain is excited at the resonance frequency using sinusoidal normal velocity perturbation introduced also at the cavity floors. The amplitude of the acoustic pressure increases as the simulation progress in time. The simulation stops when the change in the pressure amplitude from one cycle to the next is less than 1%.

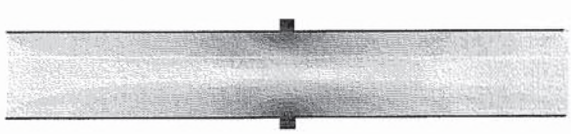

Fig. 3. Mode shape of the first transverse resonance

Figure 3 shows the first transverse mode shape at zero flow velocity in term of the contours of the acoustic pressure amplitude, as the darkness of the contours is an indication of the amplitude. As can be seen, the high pressure area is confined to the cavity. From analyzing the pressure amplitude values along the axial direction, it is observed that the pressure amplitude drops exponentially as we move away from the cavity. This is in a full agreement with the theoretical trend of trapped wave pressure drop. For the other mean flow Mach numbers, the mode shape remains similar to the zero flow velocity case.

Although the mode shape doesn't change with the velocity, the instantaneous pressure oscillation changes as a longitudinal phase difference starts to appear and increases with the Mach number. Figure 4 shows the longitudinal phase of the pressure oscillation for different Mach numbers. The midpoint of the top cavity floor is considered the reference point in the phase calculation.

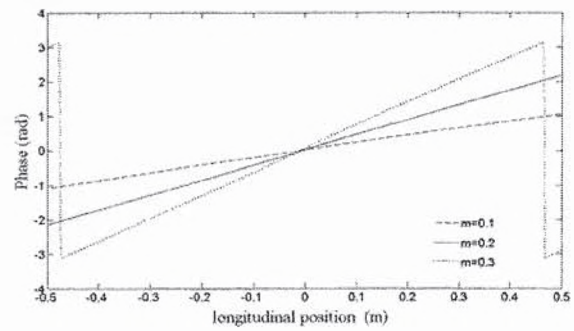

Fig. 4. Longitudinal phase change of acoustic pressure

4. CONCLUSION

This study shows that the developed CAA code is capable of simulating the convection and refraction effect of the mean flow on the acoustic resonance modes. The simulation results show that the acoustic resonance frequency decreases with the square of the mean flow Mach number. Also, it shows that the pressure mode shape doesn't change significantly in the range of the studied Mach number. However, due to the acoustic wave convection by the mean flow, a phase shift starts to develop in the steamwise distribution of acoustic pressure as the Mach number is increased.

5. REFERENCES

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