A NUMERICAL STUDY ON THE EFFECT OF VIBRATOR SHAPE ON THE DEVELOPMENT OF NONLINEAR STANDING WAVES IN A 2-D ACOUSTICAL RESONATOR

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1. INTRODUCTION

In the recent years linear and nonlinear standing acoustic waves in one-dimensional (1-D) and two-dimensional (2-D) resonators have been extensively investigated numerically, mathematically and experimentally.[1-3] To establish acoustic standing wave, we need a chamber and a vibrator. In all of the previous study, the vibrator of the acoustical resonator was assumed to be a constant shape vibrating piston. However, in the real applications different shape of the vibrator may be used to excite the resonator. In the present study the objective is to numerically analyze the effects of different shape of the vibrator on the pressure and velocity profile in 2-D nonlinear standing wave resonator.

2. METHOD

The wave equation for high-amplitude nonlinear acoustic waves in a thermo-viscous fluid is derived from the basic equations of fluid mechanics (continuity and Navier-Stokes equations) along with an appropriate state equation which can be written in 2-D as,

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0, \quad (1)$$

$$\rho\frac{Du}{Dt}+\frac{\partial p}{\partial x}=\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}, \quad (2)$$

$$\rho\frac{Dv}{Dt}+\frac{\partial p}{\partial y}=\frac{\partial\tau_{yy}}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}, \quad (3)$$

$$p=c_0^2\rho-\kappa\left(\frac{1}{c_V}-\frac{1}{c_p}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right). \quad (4)$$

where, $\frac{D(\cdot)_u}{Dt}=\frac{\partial(\cdot)}{\partial t}+u\frac{\partial(\cdot)}{\partial x}+v\frac{\partial(\cdot)}{\partial y}$, and

$$\tau_{xx}=\left(\frac{4}{3}\mu+\mu_B\right)\frac{\partial u}{\partial x}+\left(-\frac{2}{3}\mu+\mu_B\right)\frac{\partial v}{\partial y},$$

$$\tau_{yy}=\left(\frac{4}{3}\mu+\mu_B\right)\frac{\partial v}{\partial y}+\left(-\frac{2}{3}\mu+\mu_B\right)\frac{\partial u}{\partial x},$$

$$\tau_{xy}=\tau_{yx}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right). \quad (5)$$

In the above equations u and v are the velocity components, ρ is density, p is pressure, c_0 is small-signal sound speed, μ and μ_B are the shear and bulk viscosities, κ is the coefficient of thermal conduction and $\gamma=c_p/c_V$ is the ratio of specific heats at constant pressure and constant volume. The 2-D nonlinear standing wave equation can be written in the conservative form as,

$$U_t+AU_x+BU_y=CU_{xx}+DU_{yy}+EU_{xy}, \quad (6)$$

where,

$$U=\begin{bmatrix}\rho\\u\\v\end{bmatrix}, A=\begin{bmatrix}u & \rho & 0\\ c_0^2\rho^{\gamma-2} & u & 0\\ 0 & 0 & u\end{bmatrix},$$

$$B=\begin{bmatrix}v & 0 & \rho\\ 0 & v & 0\\ c_0^2\rho^{\gamma-2} & 0 & v\end{bmatrix}, C=\begin{bmatrix}0 & 0 & 0\\ 0 & \nu b/\rho & 0\\ 0 & 0 & \nu/\rho\end{bmatrix},$$

$$D=\begin{bmatrix}0 & 0 & 0\\ 0 & \nu\rho & 0\\ 0 & 0 & \nu b/\rho\end{bmatrix}, E=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & \nu(b-1)/\rho\\ 0 & \nu(b-1)/\rho & 0\end{bmatrix},$$

where ν is the kinematic viscosity and b indicates the total effect of viscosity and thermal conductivity of the fluid as well as the wall absorption, and can be obtained as, $b=2c_0^3\alpha/\omega^2\nu$, where, $\omega=2\pi f$ and α is the total absorption coefficient which is the sum of thermoviscosity absorption coefficient and wall absorption coefficient.[4] α is expressed as $\alpha=\alpha_{tv}+\alpha_{wall}$, where,

$\alpha_{tv}=\frac{\omega^2\nu}{2c_0^3}\left(\frac{4}{3}+\frac{\mu_B}{\mu}+\frac{\gamma-1}{P_r}\right)$ and $\alpha_{wall}=\sqrt{\frac{\omega\nu}{8c_0^2}}\left(1+\frac{\gamma-1}{\sqrt{P_r}}\right)\frac{p}{\Lambda}$

and $P_r=\mu c_p\kappa$ is the Prandtl number[4].

Eq. 6 is an unsteady nonlinear equation and must be solved using an appropriate numerical scheme and initial and boundary conditions. Combination of a second-order finite difference scheme and a second-order Runge-Kutta time stepping scheme provides an accurate and fast-solver numerical model which can predict pressure, particle velocity and density along the highly nonlinear standing wave resonator filled with a thermoviscous fluid with no restriction on the nonlinearity level and the type of fluid. The fluid is assumed to be initially at rest and excited by the harmonic motion of a diaphragm at $x=0$ at the frequency f (see Fig. 1). Assuming L and H to be the length and width of the tube, respectively, the following initial and boundary conditions are applicable,

$$u(0,y,t)=G(y,t),\ u(L,y,t)=v(x,0,t)=v(x,H,t)=0,$$
$$u_y(x,0,t)=u_y(x,H,t)=v_x(0,y,t)=v_x(L,y,t)=0,$$
$$\rho_x(0,y,t)=\rho_x(L,y,t)=\rho_y(x,0,t)=\rho_y(x,H,t)=0,$$
$$u(x,y,0)=v(x,y,0)=0,\ \rho(x,y,0)=\rho_0. \quad (7)$$

where $0<x<L$, $0<y<H$ and $G(y,t)$ represents shape and time-dependent excitation function of the diaphragm.

3. NUMERICAL RESULTS

To investigate the effects of the diaphragm shape on the pressure and velocity waveforms of an acoustic resonator, three different shapes are considered for the diaphragm which are, constant shape vibrating piston, circular shape and cosine shape, hereinafter referred to as cases A, B and C, respectively. These three shapes are illustrated in Fig. 1, where, u_0 is maximum velocity of the diaphragm. All simulations are conducted in air at 25^{oC} with the following thermo-physical properties, $c_0=343.4$ m/s, $\rho_0=1.2$ kg/m^3, $\nu=1.84\times10^{-5}$ N.s/m^2, $\mu_B=0.6\times\mu$ and $\gamma=1.401$. The frequency of the diaphragm is set equal to 1 kHz for all cases.

Fig. 2 depicts the variations of pressure and particle velocity over one standing wave period for u_0 equal to 0.1 and 10 m/s for cases A, B and C. At $u_0=0.1$ m/s, standing wave inside the resonator is linear, whereas, at $u_0=10$ m/s, the standing waves is nonlinear. As shown in Fig. 2, the shapes of the pressure and particle velocity are the same for the different shapes of the vibrator. However, the maximum amplitudes of pressure and velocity vary with the shape of vibrator. For the same maximum vibrational velocity of the diaphragm, the amplitudes of the pressure and velocity for constant shape vibrating piston are largest and for cosine shape are smallest.

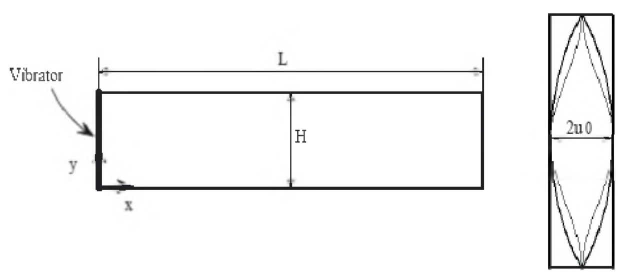

Figure 1: Schematic of the model.

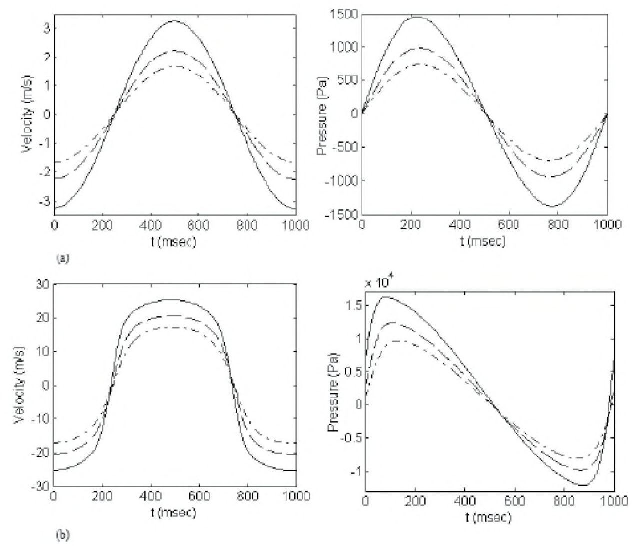

Figure 2: The variation of the pressure and particle velocity over one standing wave period for (a) $u_0=0.1$ and (b) 10 m/s. case A, solid line; case B, dash line; case C, dash-dot line.

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