INVERSE ACOUSTICAL CHARACTERIZATION OF POROUS MEDIA

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1. INTRODUCTION

Porous materials constitute a practical solution for noise abatement in many mechanical engineering applications. For this reason, several models were developed to study their acoustic behavior. These models are based on the use of viscous and thermal macroscopic parameters linked to the morphology of the material.

In this research work, the six-parameter model by Johnson-Lafarge is used to describe the viscous and thermal dissipations of acoustic waves in the porous material. The model is especially well suited to study these dissipations in the high frequency range. The six macroscopic parameters of the model are: the open porosity (ϕ), the static airflow resistivity (σ), the tortuosity (α_∞), the viscous characteristic dimension (Λ), the thermal characteristic dimension (Λ'), and the static thermal permeability (k'_0). From this choice, the technological problem one is facing is how to measure these parameters.

Several methods have been developed in the past. One can refer to references 1 and 2 for a review. The main objective of this paper is to focus on the use of the acoustical inversion technique as introduced by one of the authors [2]. This technique is only based on measurements performed in an impedance tube following standard ASTM E1050. Compared to the previous study, this inversion is not limited to only three parameters. This time, we attempt to characterize the whole set of parameters of the model. Also, the sensitivity of the method to random noise during measurements is addressed with a view to determine a confidence level.

2. METHOD

2.1 Johnson-Lafarge equivalent fluid model

Under acoustical excitations, if the frame of the porous material is assumed motionless, then the material can be modeled as an equivalent fluid. Under harmonic excitations, the Helmholtz equation governs the propagation of the compression waves in this equivalent fluid. This equivalent fluid is described by its dynamic density and dynamic bulk modulus. Both dynamic properties are related to the macroscopic parameters of the material, and depend on the frequency of excitation. In the Johnson-Lafarge model [3,4], these dynamic properties write

$$\tilde{\rho} = \rho_0 \alpha_\infty \left(1 - j \frac{\sigma\phi}{\omega \rho_0 \alpha_\infty} \sqrt{1 + j \frac{4\alpha_\infty^2 \eta \rho_0}{\sigma^2 \phi^2 \Lambda^2} \omega} \right)$$

and

$$\tilde{K} = \frac{\gamma P_0}{\gamma - (\gamma - 1)\left(1 - j \frac{8\eta}{\omega \Pr \Lambda'^2 \rho_0} \sqrt{1 + j \frac{\omega \Pr \Lambda'^2 \rho_0}{16\eta}} \right)^{-1}}$$

where ρ_0, η, and γ are the density, viscosity, and specific heat ratio of the saturating air, Pr is the Prandtl number, and P_0 is the static pressure.

From these two dynamic properties, one can deduce the normal incidence acoustic surface impedance of a porous sample of thickness d by

$$Z_{sn}^{\text{model}} = -j \cdot \frac{\sqrt{\tilde{\rho}\tilde{K}}}{\phi} \cdot \cot\left(\omega \sqrt{\frac{\tilde{\rho}}{\tilde{K}}} d \right).$$

2.2 Inverse technique of characterization

The characterization following the proposed inverse technique consists in computing the difference (or residual) between the experimental data and the data predicted by the model. Then, the unknown macroscopic parameters are adjusted so that the residual is minimized..

Consequently, the difficulty of the problem is to correctly minimize the following residual function by using an algorithm of descent:

$$\mathrm{R}\left(\mathbf{X}\right) = Z_{sn}^{\text{model}}\left(\mathbf{X}\right) - Z_{sn}$$

with

$$\mathbf{X} = \left\{ \phi, \sigma, \alpha, \Lambda, \Lambda', k'_0 \right\}$$

In a first time, R is linearized around an initial point $\mathbf{X_0}$ thanks to the first order Taylor formula. Defining the jacobian matrix of R by H, this yields:

$$R\left(\mathbf{X}\right) = R\left(\mathbf{X}_0\right) + \mathbf{H} \times d\mathbf{X}$$

By solving this equation, $d\mathbf{X}$ is found, and a new point is deduced. The operation is repeated until $d\mathbf{X}$ tends to zero - this corresponds to a minimum of R, and to the corresponding set of parameters.

3. RESULTS

Initially, the strategy to test the inverse method was not considering noise in the measurement of Z_{sn}. In this noiseless case, the results were very conclusive for a broad range of materials. The range of the material properties are given in Table 1.

When random noise is considered in Z_{sn}, the conclusion is different for some materials. The most important impact of noise on the inversely found parameters occurs for the thermal characteristic length. This parameter was found to be very sensitive to noise, more especially when its true value is large. This is shown in Figure 1, where the error on the found Λ' is plotted in function of the error on Z_{sn} for different values of Λ'. One can note the inversion procedure yields a larger error (always underestimate) on Λ' as the error on Z_{sn} increases. This error increases much more rapidly than the error on Z_{sn} and as Λ' is larger.

4. DISCUSSION

From Figure 1, one can conclude that it may be difficult to obtain a good estimate of the thermal characteristic length with an acoustical inverse method if noise is not sufficiently controlled. To obtain a higher degree of accuracy on Λ' using the inverse approach, it is necessary to base the inversion procedure on an acoustical measurement that is more sensitive to the thermal characteristic length. More indications will be given at the conference.

REFERENCES

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Table 1 : Range of parameters

Parameter	Lower bound	Upper bound
ϕ	0.6	1
σ	3000 Ns/m^4	100000 Ns/m^4
α	1	7
Λ	15 µm	500 µm
Λ'	15 µm	500 µm

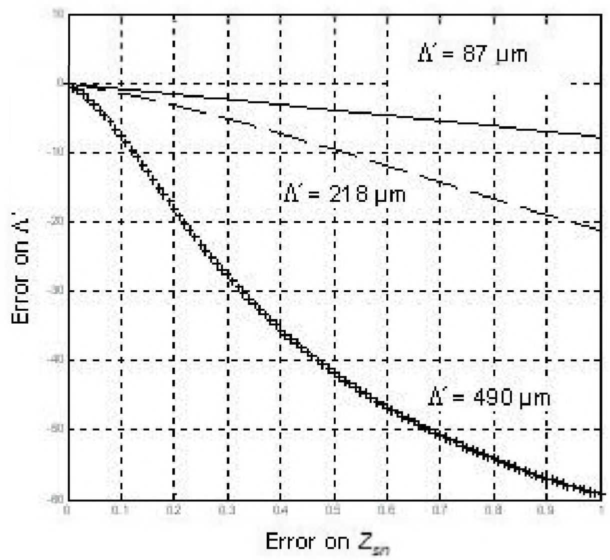

Figure 1 : Error on inversely found Λ' in function of the error on Zsn