Modeling of general laminate composite structures with viscoelastic layer

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1. INTRODUCTION

Damped multilayer structures such as flat panels are largely used in automotive and aerospace constructions. Most papers in the field treat isotropic structures[1] with damped layers or patches in symmetrical or asymmetrical sandwich configurations. The increasing requirements of accuracy have encouraged important improvements[1-3] in the modeling approaches. The industry development of new multilayer damped configurations (not necessarily of a symmetric sandwich nature) favors the use of general laminate models[2]. Moreover, the new iterative identification algorithms, used for the characterization of the viscoelastic materials' dynamic properties, require a tremendous amount of computational effort[3]. These methods[3] update successively the complex eigenvalues and eigenvectors until required accuracy is achieved. It is worth pointing out that accurate and fast numerical solutions are imperatives for such applications.

This paper describes the modeling of reasonably thick general composite laminate plates and beams with linear viscoelastic damping. The principal aim is the fast and accurate modeling of such structures for low to high frequencies. The problem is solved by discrete laminate method in a wave approach context. The discrete laminate approach assumes each layer described by a Reissner-Mindlin displacement field which leads to equilibrium relations accounting for membrane, transverse shearing, bending and full inertial terms. Each layer accounts for orthotropic plies orientations. The discrete description of each layer allows for accurate handling of thin/thick laminates and sandwich panels over the audible frequency range. In particular, at high frequencies the combination of (i) propagating wavelength characteristics (short wavelengths) and (ii) the layout's physical properties (certain layers are much stiffer than adjoining ones) may result in decoupled out-of-phase movement of stiff layers. Such phenomena are correctly captured by the discrete laminate approach.

2. THEORY

This study deals with layouts of an unlimited number of composite and viscoelastic layers. Figure 1 represents the global geometrical configuration of a composite panel (Figure 1.a) and a composite beam (Figure 1.b) with side dimensions L_x and L_y and total thickness h. The layered construction is considered, in general, asymmetrical. The origin of the coordinates system is defined on a reference surface passing through the middle thickness as represented in Figure 1.

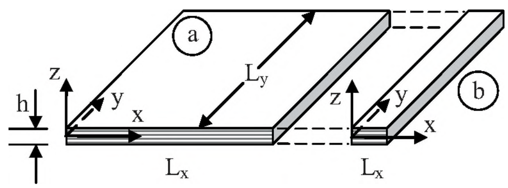

Figure 1. Global geometrical configuration. Flat laminated composite panel (**a**), and laminated composite beam (**b**) of L_x and L_y side dimensions and h total thickness.

Membrane and bending displacements as well as shearing rotation are generally expected to act in each layer; the displacement field of any i^{th} discrete layer of the panel is of Mindlin's type.

The resultant stress forces and moments of any layer are defined in Ref. [5]. There are three interlayer forces along x, y, and z directions between any two layers. Consequently, the total number of interlayer forces is $3(N-1)$ where N is the number of layers.

For any i^{th} layer there are five equilibrium equations:

$$\begin{aligned}
&N^i_{x,x} + N^i_{xy,y} + F^i_x - F^{i-1}_x = m^i_s u^i_{,tt} + I^i_{z2}\varphi^i_{x,tt}\\
&N^i_{xy,x} + N^i_{y,y} + F^i_y - F^{i-1}_y = m^i_s v^i_{,tt} + I^i_{z2}\varphi^i_{y,tt}\\
&Q^i_{x,x} + Q^i_{y,y} + F^i_z - F^{i-1}_z = m^i_s w^i_{,tt}\\
&M^i_{x,x} + M^i_{xy,y} - Q^i_x + z^i F^i_x - z^{i-1}F^{i-1}_x = I^i_z\varphi^i_{x,tt} + I^i_{z2}u^i_{,tt}\\
&M^i_{xy,x} + M^i_{y,y} - Q^i_y + z^i F^i_y - z^{i-1}F^{i-1}_y = I^i_z\varphi^i_{y,tt} + I^i_{z2}v^i_{,tt}
\end{aligned} \tag{1}$$

Rotational inertia, in-plane, bending as well as transverse shearing effects are accounted for in each layer. Also, orthotropic ply's directions are used for any lamina composing a layer. The expressions of the transverse shear stress forces Q_i, the in-plane stress forces N_i, the inertial terms I_i, and the stress moments M_{ij} of each layer are defined in Ref. [5].

2.1. Dispersion relation

For any layer, the dynamic equilibrium equations are rewritten using equations (1) with appropriate algebraic manipulations and has $5N+3(N-1)$ variables regrouped in a hybrid displacement-force vector $\langle e \rangle$. Next, the system is expressed in the form of a generalized polynomial complex eigenvalue problem:

$$k_c^2[A_2]\{e\} - jk_c[A_1]\{e\} - [A_0]\{e\} = 0; \tag{2}$$

where, $k_c = \sqrt{k_x^2 + k_y^2}$, $j = \sqrt{-1}$ and [A$_0$], [A$_1$], [A$_2$] are real square matrices (in the absence of damping) of dimension $5N+3(N-1)$ defined in Ref [5].

Relation (2) has $2(5N+3(N-1))$ complex conjugate eigenvalues and represents the dispersion relations of the laminated composite structure. The matrices in relation (2) become complex when viscoelastic layers compose the layout.

The pure arithmetically real solution with the highest amplitude corresponds, in the case of thin isotropic structures, to the bending wavenumber. This solution has three asymptotical behaviors for sandwich configurations: pure bending at low frequencies, core's transversal shearing at mid-frequencies and pure bending of skins at high frequencies.

In the following, the first propagative solution (of highest amplitude) is retained and used to illustrate applications of the proposed model. This propagative solution corresponds to transversal displacements motion (bending for the thin structures' case) accompanied by in-plane and transversal shearing internal deformations.

2.2. Equivalent loss factor

The propagating solutions and the associated eigenvectors of the relation (2) are used to express the strain energy U_n of the hybrid problem. Next, the equivalent loss factor of a panel or beam with N layers, associated to the n^{th} propagating wave is expressed as:

$$\eta_n = \frac{2}{\pi}\int_0^{\pi/2} \frac{\sum_{k=1}^{N_{lay}} \eta_k U_n^k}{U_n} d\varphi = \frac{2}{\pi}\int_0^{\pi/2} \frac{\sum_{k=1}^{N_{lay}} \eta_k v_n^T Re[\lambda_n A - B] v_n}{v_n^T Re[\lambda_n A - B] v_n} d\varphi \qquad (3)$$

where, the matrices A and B are given in Ref. [5].

Relation (3) compute the total damping loss factor of a composite laminate as the average of the angular distributions of the damping loss factor over a quart of the wavenumber space with respect to the heading directions.

2.3. Numerical results and validation

Examples of comparisons to experimental and spectral finite elements results are presented and discussed in this section. Figure 2 presents the comparisons between experimental results[6] and the present discrete laminate approach. Excellent agreement is observed. The resonance frequencies and the resonance amplitudes of the first four modes are accurately estimated.

The next configuration concerns very thin laminated steel beam with a constrained viscoelastic layer, representative of laminated steel used in automotives. Damping loss factor is computed using the present approach and spectral finite element model presented in Ref. [2], and the results are plotted in Figures 3. Excellent agreement is observed in Figure 3 between the analytical discrete laminate and spectral finite elements approaches.

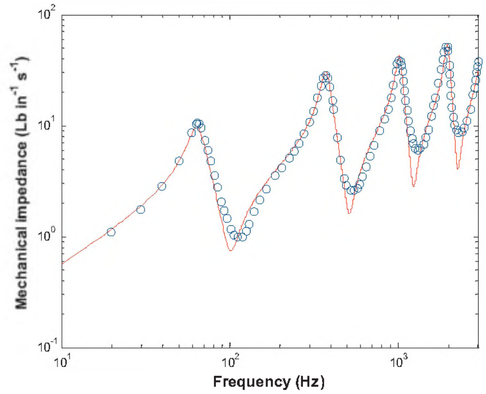

Figure 2. Driving point mechanical impedance of a sandwich beam (Sun and Lu[6], Fig. 4.3, Pag.173). Experimental validation: (—) Discrete laminate; (○○) Experimental.

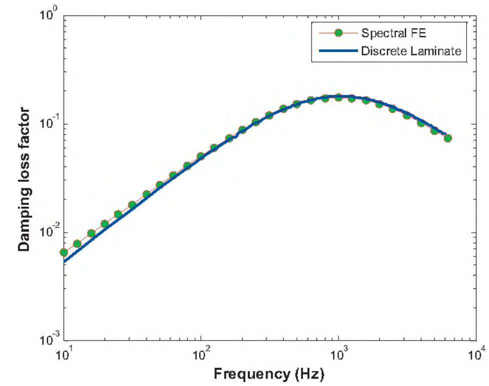

Figure 3. Damping loss factor of a cantilever sandwich beam.

3. CONCLUSIONS

The modeling of thick composite laminated plates and beams with linear viscoelastic damping layers was described. A theoretical approach has been developed so as to fulfill a present need for fast and accurate numerical models generally dedicated to optimization and inverse characterization applications. The problem was solved by analytical discrete laminate method in a wave approach context. The model has been successfully validated with experimental and numerical results. Moreover, the model was applied to the calculation of the structural loss factor associated to the bending wave-type.

References:

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