Symmetry of Viscous Permeability Tensor in Porous Media

C. Perrot, F. Chevillotte, and R. Panneton
GAUS, Department of Mechanical Engineering, Université de Sherbrooke, Canada, (Qc) J1K 2R1

1. INTRODUCTION

This paper reports parts of recent simulation results [1] carried out in order to illustrate a rather theoretical and somewhat non intuitive property of the viscous permeability tensor in porous media: considering periodic porous structures in the long wavelength limit, even if the structure is anisotropic, its acoustical properties will remain isotropic, i.e. : the velocity fields are qualitatively different in principal directions of a porous structure showing different local arrangements to an incident acoustic wave; however, they remain identical on average. The practical importance of this property is that it can be used to assess the validity and accuracy of numerical simulations in the emerging field of bottom-up approaches for microstructure optimization of sound absorbing materials. This paper is organized as follows. In Sec. 2 the basic equations used to describe the physical properties of a porous material are presented at the macroscale of day-to-day engineering applications. In Sec. 3 a short description of the permeability tensor symmetry property is given from the standpoint of the equations governing the physics at the pore scale, i.e. microscale. In Sec. 4, numerical results in porous structures with hexagonal symmetry are presented to illustrate the previous property. Good agreement between theory and numerical experiment is found.

2. MACROSCOPIC DESCRIPTION

This Sec. deals with the classical description of the flow of a viscothermal fluid in a motionless homogeneous porous structure. The statistical properties of the porous frame can be defined in homogenization volumes with dimensions much smaller than the wavelength of the acoustic waves that propagate in the saturating fluid. The microscopic quantities that describe the flow (pressure p, velocity $\mathbf{v}$) present variations at the microscopic scale in the homogenization volume. To smooth out these variations and leave only the macroscopic variations, fluid-phase average is introduced (< >). At a given frequency, a fundamental relation called the generalized Darcy law in memory of its pioneering work [2] and defined as $\Phi\langle \mathbf{v}_i(\omega)\rangle = -[k_{ij}(\omega)/\eta]\,\mathbf{grad}_j\langle p\rangle$, is linking the gradient of the macroscopic pressure to the macroscopic velocity. Here, Φ is the open porosity, η is the viscosity of the fluid, and $k_{ij}(\omega)$, previously defined and studied by Johnson et al. [3], is the second order dynamic viscous permeability tensor only depending on frequency and on the geometry of the porous structure.

3. PERMEABILITY TENSOR SYMMETRY

As discussed in Ref. [4], describing the periodic oscillating flow created in a porous medium by an external unit harmonic pressure gradient, one has to solve in the fluid volume Ω_f the following set of scaled equations (unsteady Stokes problem):

$$\frac{-i\omega}{\nu}\mathbf{w} = -\nabla\pi + \Delta\mathbf{w} + \mathbf{e} \quad \text{in } \Omega_f, \tag{1}$$

$$\nabla\cdot\mathbf{w} = 0 \quad \text{in } \Omega_f, \tag{2}$$

$$\mathbf{w} = 0 \quad \text{in } \partial\Omega, \tag{3}$$

where $\mathbf{e}$ is a unit vector, $\nu = \eta/\rho_0$, ρ_0 is the air density at rest, and $\mathbf{w} = \mathbf{v} / |\mathbf{grad}\langle p\rangle / \eta|$ is the scaled velocity field (in m^2). The solution to the problem (1)-(3) is fixed by adding the condition that π is a spatially stationary or periodic field. This problem is relevant to sound propagation as long as the wavelength is large enough for the saturating fluid to behave as an incompressible fluid in volumes of the order of the homogenization volume (a period in the case of periodic structure).

The demonstration proposed in Ref. [1] states that, by using three individual solicitation vectors $\mathbf{e}^i$ in three perpendicular directions, with components $\mathbf{e}_{ij} = \delta_{ij}$, $k_{ij}(\omega)$ can be written in a symmetrical form in i and j :

$$k_{ij}(\omega) = -\frac{i\omega\phi}{\nu}\left\langle \mathbf{w}^j\cdot\mathbf{w}^i\right\rangle + \phi\left\langle\left(\frac{\partial}{\partial x_m}\mathbf{w}^j\right)\cdot\frac{\partial}{\partial x_m}\mathbf{w}^i\right\rangle. \tag{4}$$

This shows that the viscous permeability tensor is symmetric, $k_{ji} = k_{ij}$. As a consequence, there exists a system of orthogonal axes, the principal axes, where the tensor has only diagonal elements different from zero. If the medium presents the trigonal, the tetragonal, and the hexagonal symmetry, the axis of symmetry Z must coincide with one of the principal axes and the invariance of the system through some discrete rotations along this axis necessarily means that the two transverse eigenvalues of the tensor are the same (transverse isotropy). In particular, the static permeability k_{0ij} has value k_{0Z} along axis Z, and unique values k_{0P} for all directions orthogonal to Z.

$$\begin{bmatrix} k_{011} & k_{012} \\ k_{021} & k_{022} \end{bmatrix} = \phi \begin{bmatrix} \left\langle \mathbf{w}_{01}^{1} \right\rangle & \left\langle \mathbf{w}_{01}^{2} \right\rangle \\ \left\langle \mathbf{w}_{02}^{1} \right\rangle & \left\langle \mathbf{w}_{02}^{2} \right\rangle \end{bmatrix} = \begin{bmatrix} 4.9420 & 0 \\ 0 & 4.9353 \end{bmatrix} \times 10^{-8}\,\mathrm{m}^2 \qquad (5)$$

4. NUMERICAL SIMULATIONS

Parameters k_0 is important for the prediction of the acoustic properties of porous media. The constraint which exists when the structure has the uniaxial symmetry can provide a test for the precision of the simulations. An example is given in that Sec. where the static permeability k_{0P} is evaluated in a hexagonal porous structure. As an illustration, we show in Fig. 1 the two components of the static scaled patterns obtained for excitation along the two principal directions in the plane perpendicular to the axis of symmetry of the periodic geometry. This yields the tensor written in Eq. (5). The relative differences found for the horizontal and vertical directions are less than 0.13 %. Also, the non-diagonal terms are numerically equal to zero. These results are consistent with the theory presented in Section 3, and, as a consequence, prove the validity of our numerical implementation. Finally, these results provide, in the limit of the precision of the finite element method, a numerical illustration of the symmetry property of the viscous permeability tensor reported in Section 3. In a plane perpendicular to the axis of a porous material with hexagonal symmetry, viscous permeability tensor is reduced to a constant diagonal element.

5. CONCLUSION

In conclusion, a simple proof has been proposed of a somewhat non-intuitive property of the dynamic viscous permeability tensor. As long as the wave length is much larger than the pore sizes, for periodic porous structures with hexagonal symmetry presenting different local configurations to a wave propagating in different directions in the plane perpendicular to the axis of symmetry, the permeability tensor is diagonal and constant in the different directions. This property can notably be used for error estimation of numerical computation.

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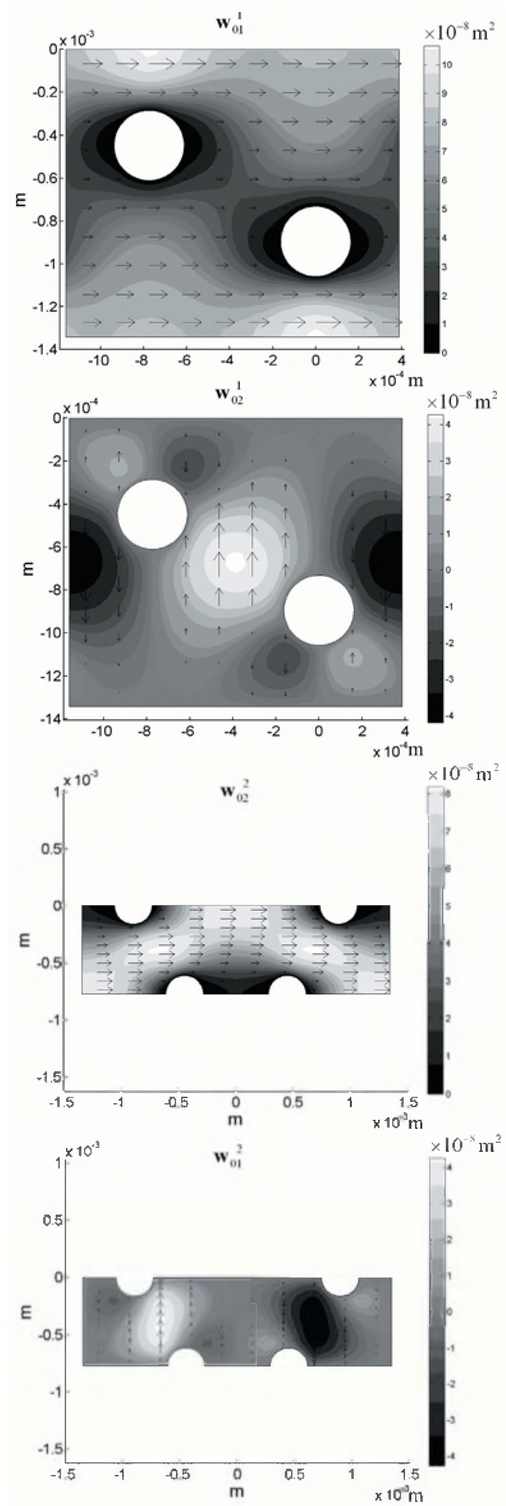

Fig. 1. Static scaled velocity fields obtained by solving the steady-Stokes problem in principal directions of an hexagonal periodic porous structure.