ULTRASOUND TRANSMISSION THROUGH TIME-VARYING PHONONIC CRYSTALS

Derek W. Wright[1] **and Richard S.C. Cobbold**[1]
[1]Institute of Biomaterials and Biomedical Engineering, University of Toronto, 164 College St, Ontario, Canada M5S3G9

1. INTRODUCTION

Over the past few years there has been a substantial growth of interest into new engineered materials called metamaterials, which include photonic crystals. These materials are designed to manipulate electromagnetic waves, such as microwaves and light, in new ways that enable properties and effects not found in natural materials. This has stimulated major interest in the development and understanding of their acoustic wave analogue, namely phononic crystals [1][2]. Phononic crystals manipulate acoustic waves through scattering to realize numerous unique effects. In particular, it may be possible to achieve sub-wavelength imaging, which has already been predicted and experimentally demonstrated for electromagnetic metamaterials [3][4], and has been predicted for certain photonic crystals [5].

Phononic crystals are artificial crystal arrangements of materials with differing acoustic properties. The incident acoustic waves scatter off of the material interfaces present in the crystal lattice, interfering constructively or destructively. Accordingly, the wavelengths of interest are those comparable to the spacing of the crystal lattice. Some of the effects already experimentally demonstrated in certain phononic crystals are transmission band-gaps [6], negative refraction [7], ultrasound focusing in 2D [1] and 3D [8], and phonon tunnelling [9].

The acoustic properties of a phononic crystal are fixed by its design parameters, such as the crystal lattice configuration and spacing, and the materials used. Once a crystal has been fabricated it is not possible to change its acoustic properties, resulting in a static crystal whose usefulness is limited to its designed operating parameters.

We seek to control the acoustic properties of a phononic crystal by changing its material parameters as a function of time, which will dynamically affect the acoustic wave scattering within the crystal. This may enable time-variant phononic crystals which would have adjustable acoustic properties depending on the nature of the material parameter variation. In order to determine how material parameter variation will affect the acoustic properties of a phononic crystal, we have extended the existing static scattering theory to handle time-varying material parameters. We wish to examine the resulting governing equations to gain insight into the factors that affect acoustic transmission through time-varying phononic crystals.

2. METHOD

2.1. Our new method

We have used the 1D transmission matrix method (TMM) as a starting point since it gives a closed-form solution to wave propagation in periodic media [10]. The existing TMM theory was then expanded to accommodate time-varying material parameters, which we call the *time-varying transmission matrix method* (TV-TMM). Incident waves within a time-varying phononic crystal are modulated at every scattering interface. This continuous interaction between the propagating waves and the time-varying parameters dramatically affects the nature of wave transmission through a phononic crystal. The resulting closed-form expressions can provide insight into how the time-varying material parameters affect wave propagation.

The finite-difference time-domain (FDTD) method [11] can also simulate time-varying phononic crystals, however, it gives no analytical insight into *why* a particular result occurs – only *what* occurs. Moreover, because the accuracy depends on the resolution in the space and time domains, better results take longer to generate. The FDTD method was thus used as a benchmark to verify our new method.

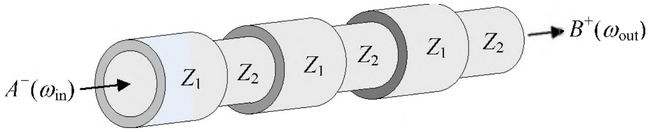

Fig 1. A corrugated tube waveguide. The incident waves are scattered at every impedance discontinuity, which occur between each tube section. $A^+(\omega_{in})$ and $B^+(\omega_{out})$ are the input and transmitted spectra, respectively.

2.2. A representative 1D system

A corrugated tube waveguide is a tube in which the diameter of the tube changes periodically along its principle axis as shown in Figure 1. Acoustic wave propagation within such a corrugated tube waveguide is analogous to acoustic wave propagation with a 1D phononic crystal consisting of alternating layers of differing media. Each tube section of a given diameter has particular acoustic impedance, Z. Thus, the alternating diameter tube shown in Figure 1 has two acoustic impedances: Z_1 and Z_2.

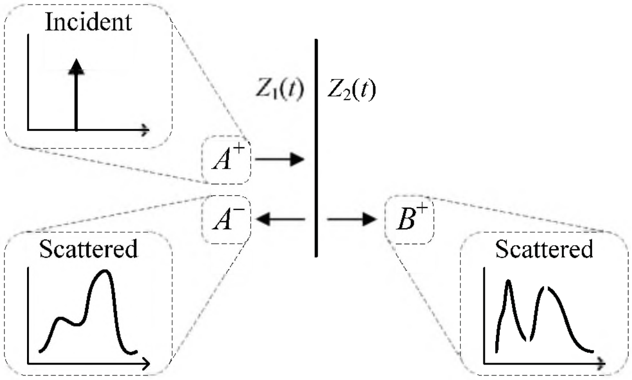

Fig 2. An illustration of how an incident monotone (A^+) is scattered and modulated (A^- and B^+) at every time-varying impedance discontinuity. The inset plots represent frequency-domain spectra.

In a corrugated tube waveguide, when an incident acoustic wave meets an impedance discontinuity the wave is scattered, meaning that some of the energy is reflected and the remainder is transmitted. In the static case, the transmission and reflection coefficients are simply constants, so that the scattered waves have the same frequency content as the incident wave. However, in the dynamic case the impedances are time-varying due to material parameter variation, thus the transmission and reflection coefficients are also time-varying. The incident wave is modulated by the time-varying transmission and reflection coefficients resulting in a spectrum of scattered waves. Such a situation is illustrated in figure 2 for an incident monotone.

3. RESULTS

3.1. Verification

Using the 1D tube waveguide from [12] as a representative model of a 1D phononic crystal, transmission equations for both continuous (including non-periodic) and periodic material parameter variations were derived. The new TV-TMM formulation matches the predictions of our FDTD simulator. Furthermore, the TV-TMM simulation execution times were four to five orders of magnitude faster than the FDTD simulations used for comparison and provide exact solutions at the frequencies of interest.

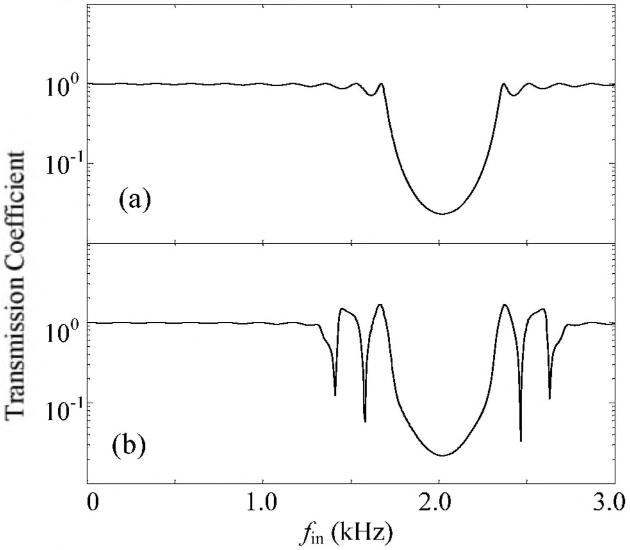

Fig 3. Fundamental frequency transmission coefficients for ten (a) static and (b) time-varying segments. In (b), the parameter variation frequency is 500 Hz and the second tube cross-sectional area is varying by ± 5%.

3.2. Results

The fundamental frequency transmission coefficient for the ten tube segments is shown for both the static and time-varying cases in figure 3, generated by varying the input frequency and plotting the output magnitude of that same frequency. Figure 3a shows the transmission band-gap characteristics centred at 2 kHz in the static case. In figure 3b, the transmission spectrum now includes energy that has been modulated to and from the harmonics by the time-varying transmission and reflection coefficients. Notice that the shapes of the band-gap and band-gap edges have been significantly altered when compared to the static case. This raises the possibility of changing the band-gap characteristics using material parameter variation as the controlling mechanism. One could envision a controllable band-pass or notch filter using such an arrangement.

4. DISCUSSION

Our new method for predicting the transmission properties of time-varying phononic crystals gives closed-form solutions to the transmission through single and multiple time-varying phononic crystal segments. Using this method, we predict that the transmission properties of a time-varying phononic crystal can be made dramatically different from a similar static phononic crystal.

Currently, we are analyzing the resulting equations for acoustic wave transmission through time-varying phononic crystals provided by our new method. We hope to derive a better understanding of the relationship between the nature of the material parameter variation and the resulting output spectra. Furthermore, two and three dimensional phononic crystals have the additional property of acoustic wave refraction, a fact which leads us to ask what effect material parameter variation may have on imaging through these crystals.

To help us answer these questions, we intend to extend our 1D TV-TMM formulation into 2D by applying a similar approach to that used in 2D multiple scattering theory [13]. The extension from 2D to 3D should also be possible since 2D and 3D multiple scattering theories are quite similar in form.

REFERENCES

[1] A. Sukhovich, L. Jing & J. H. Page. (2008). Negative refraction and focusing of ultrasound in two-dimensional phononic crystal, Phys. Rev. B 77 014301

[2] J. H. Page, A. Sukhovich, S. Yang, M. L. Cowan, F. Van Der Biest, A. Tourin, M. Fink, Z. Liu, C. T. Chan & P. Sheng. (2004). Phononic crystals Phys. Stat. Sol. (b) 241 3454–62

[3] J. B. Pendry. (2000). Negative refraction makes a perfect lens Phys. Rev. Lett. 85 3966–9

[4] A. Grbic & G. V. Eleftheriades. (2004). Overcoming the diffraction limit with a planar left-handed transmission-line lens Phys. Rev. Lett. 92 117403

[5] F. AbdelMalek, W. Belhadj & H. Bouchriha. (2005). FDTD study of subwavelength imaging by a photonic crystal slab Photonics Nanostruct. Fundam. Appl. 3 19–24

[6] M. S. Kushwaha & P. Halevi. (1994). Band-gap engineering in periodic elastic composites Appl. Phys. Lett. 64 1085–7

[7] X. Zhang & Z. Liu. (2004). Negative refraction of acoustic waves in two-dimensional phononic crystals Appl. Phys. Lett. 85 341–3

[8] Yang S., Page J. H., Liu Z., Cowan M. L., Chan C. T. & Sheng P. (2004). Focusing of sound in a 3D phononic crystal Phys. Rev. Lett. 93 024301

[9] S. Yang, J. H. Page, Z. Liu, M. L. Cowan, C. T. Chan & P. Sheng. (2002). Ultrasound tunneling through 3D phononic crystals Phys. Rev. Lett. 88 104301

[10] C. E. Bradley. (1994). Time harmonic acoustic Bloch wave propagation in periodic waveguides. Part I. Theory J. Acoust. Soc. Am. 96 1844–53

[11] Y. Tanaka, Y. Tomoyasu & S-i Tamura. (2000). Band structure of acoustic waves in phononic lattices: Two-dimensional composites with large acoustic mismatch Phys. Rev. B 62 7387–92

[12] X. Hu, Z. Hang, J. Li, J. Zi & C. T. Chan. (2006). Anomalous Doppler effects in phononic band gaps Phys. Rev. E 73 015602(R)

[13] P. Martin. (2006). Multiple Scattering (New York: Cambridge University Press)

ACKNOWLEDGMENTS

We wish to acknowledge financial support from the Natural Sciences and Engineering Research Council of Canada and an Ontario Graduate Scholarship to DWW.