MEASUREMENT AND CALCULATION OF THE PARAMETERS OF SANTUR

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1.   INTRODUCTION

This paper, concerns the instrument, Santur and measurement and calculation of its physical characteristics and acoustical properties such as pitch deviation and inharmonicity factor. These parameters provide a better understanding of the instrument.

The Santur

The Santur (Fig.1-a), is a flat string instrument, which is played with a pair of hammer sticks. Santur originated in Iran and is known as a Hammered Dulcimer in English. It is a direct ancestor of Piano.

The pair of hammer sticks (Fig.1-b), are held between the index and the middle fingers and are used to hit the strings. When the notes are played, a small deflection of the strings creates a loud voice. Sticks are usually coated by a felt. The impact, makes it thinner and harder through time.

Four strings are vibrated for each note. They are stretched on the sound board, pulled between the string holders (Fig.2-a) and the tuning pegs (Fig. 2-b, Fig. 3-b), and sit on a bridge between these two ends (Fig.3-a).

The strings hinge on the left and right edges of the soundboard. The notes are adjusted by the tuning pegs, using a tuning key (Fig.1-c). It is also used as a hammer to hit the tuning pegs. The bridges are movable and can continuously change the pitch by several whole steps.
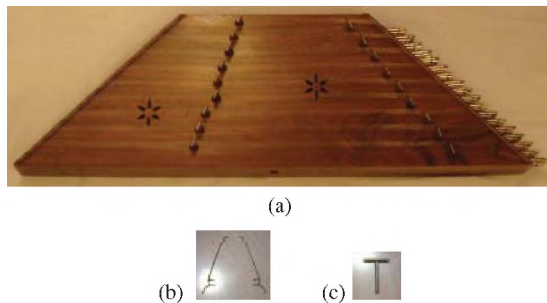

(a)

(b)   (c)

Fig. 1 a) Santur b) Sticks (Mezràb) c) Tuning Key

Parallel sides of a 9-bridge[1] Salari[2] Santur, on which we did the measurements, are 90.5cm and 34.8cm. The other sides (left and right) are 38.9cm and 39.0cm. The top and the back plates[3] are 6.4cm apart. Their thicknesses are 5.5mm and 8.0mm respectively. The lengths of the four strings of a note are not exactly the same. They are between 36.8 and 37.8cm for F5-F6 and between 84.8cm and 86.25cm for C3.

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[1] Modern Iranian Santurs usually have 9 bridges. 11 and 12-bridge Santurs are also prevalent [1].

[2] Master Daryoush Salari is a famous Santur producer in Iran. He has adapted innovative production techniques.

[3] Upper and lower boards
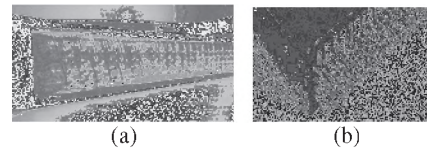

(a)   (b)

Fig. 2 a) String holders b) tuning pegs

Diameters of strings are between 0.35-0.36mm depending on the string's age and the tension. A bridge has a height of 2.3cm, and there is a metal roll of diameter 2.5mm on top of it (Fig. 3-a). The length of the strings between the right side of top plate and the tuning pegs (Fig. 3-b) is 2-6cm for the first and fourth strings.
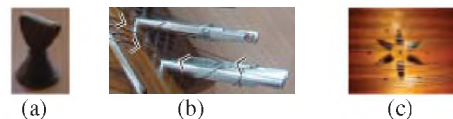

(a)   (b)   (c)

Fig. 3 a) Bridge with a metal roll b) strings between top plate edge and tuning pegs c) Sound hole

Four pieces of flat wood along with some sound posts (rigid wooden bars) support the top plate over the back plate. They bear the force exerted by bridges, string holders and tuning pegs. The resonant body of a Santur is hollow, but the sound posts keep the instrument from collapsing. The two sound holes (Fig. 3-c) are of diameter 5cm. They influence the timbre and serve to enhance the sound quality. The tone range is: E3 (164.8 Hz)-F6 (1396.9 Hz), while the first bass note is usually tuned at C3 (130.8 Hz), instead of E3.

2.   PITCH AND HARMONIC DEVIATION

Due to inharmonicities, we expect the overtones of a fundamental frequency (f_0) to move slightly upwards. The positions of overtones for a stiff string is calculated by [2]:

$$f_h = h f_0 \sqrt{1 + \beta h^2} \qquad (1)$$

Where, f_h is an overtone, h is harmonic index, and β is inharmonicity factor[4]. Therefore, f_0 is slightly shifted to $f_0^1 = f_0 \sqrt{1 + \beta}$ and Eq. 1 can be rearranged as:

$$f_h = h f_0^1 \frac{\sqrt{1 + \beta h^2}}{\sqrt{1 + \beta}} \qquad (2)$$

Where f_0^1 is the measured fundamental frequency.

Different factors such as the thickness, length and the string tension contribute to the inharmonicities. Increasing the thickness and the tension force or decreasing the string's length result in a higher inharmonicity factor. The following equation describes the inharmonicity factor of a

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[4] β is around 0.0004 for Piano [3] and 0.00031 for Santur.

string in term of its length, l, diameter, d, and tension, T [2, 3]:

$$\beta = \frac{E\pi^3 d^4}{64 l^2 T} \qquad (3)$$

E is the Young's module[5]. Young's module is the constant of elasticity of a substance.

3. RESULTS

Our dataset consists of 10 isolated samples per note. The frame size is 32768 samples and with a sampling rate of 44.1 kHz, frequency resolution becomes 1.35 Hz. The positions of f_0 and its overtones can be used to calculate the inharmonicity factor.

The pitch deviation is measured for the following notes[6]: F3, Aq3, C4, F4, Aq4, C5, F5, Ab5, C6, Eb6. It was observed that the first and second octaves are compressed by 11 and 28 cents respectively, while the third octave is stretched by 20 cents. Except an F4 sample which has a lower pitch deviation and is interpreted as miss-tuned, the bass and middle pitches tend to be less than the tempered values as we move towards higher notes, while the treble pitches tend to be more. Thus, the treble pitches on a Santur are stretched similar to Piano [4], while the bass and middle pitches are compressed in contrast.

Then, the harmonic deviation of the first 8 overtones from multiples of f_0^1 is calculated. Using Eq.2, the inharmonicity factor[7], β can be calculated in terms of the f_0^1 and overtone positions f_h:

$$\beta = \frac{(f_h/hf_0^{1})^2 - 1}{(h^2 - (f_h/hf_0^{1})^2)} \qquad (4)$$

Or in terms of the hth and m^{th} overtones, f_h and f_m:

$$\beta = \frac{\left(\frac{m.f_h}{h.f_m}\right)^2 - 1}{\left(h^2 - m^2\left(\frac{m.f_h}{h.f_m}\right)^2\right)} \qquad (5)$$

The inharmonicity factors of different notes are not the same. As we move towards higher notes, the inharmonicity factor increases regardless of the tone area (bass, middle and treble). The values calculated through different harmonics are also different. We will calculate the average value over different notes and different harmonics.

It should be noted that variations of the inharmonicity factor, using the first few harmonics are high. So, it is generally better to use higher harmonics in the calculations [3]. This improves the accuracy of calculations due to frequency resolution as well.

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[5] Young's module represents the ratio of stress to strain for a string or a bar of the given substance. It is the force per unit cross section of a material divided by the increase in its length resulting from the force.

[6] "q" shows half-flat and "b" shows flat [1].

[7] Here we ignore the impedance of the bridges and the sound board.

We have encountered 8 overtones here. So, calculation of the inharmonicity factor based on h8 and h4 or h5, might be a good choice to avoid using close partials. Fig. 4 shows the inharmonicity factor, calculated based on the positions of the 8th overtone and 1st to 7th overtones. The curve with considerable changes at C4, Aq4, Eb5 and Ab5 corresponds to the measurement through neighboring h8 and h7 harmonics and will be excluded from our calculations. The average value of inharmonicity factor for the 9-bridge Santur is 0.00031.
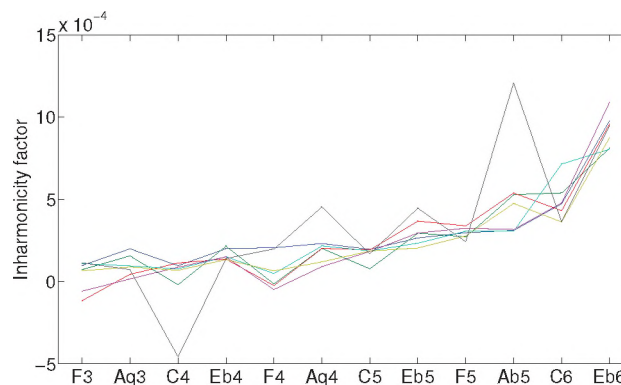

Fig.4 Inharmonicity factor based on the 8th overtone vs. 1st to 7th overtones

4. CONCLUSION

In this paper, the Santur instrument and its parameters were explained. The treble pitches on a Santur are stretched similar to the notes on a Piano, while the bass and middle pitches are compressed in contrast. The inharmonicity factor for a 9-bridge Santur is calculated, based on the average over different notes and overtones. It is 0.00031.

Future work will be on determining a more accurate value for inharmonicity factor by considering other notes, higher harmonics and analysis of the sound of other Santurs.

5. REFERENCES

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