A SEMI-ANALYTICAL APPROACH TO THE STUDY OF THE TRANSIENT ACOUSTIC RESPONSE OF CYLINDRICAL SHELLS

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1. INTRODUCTION

A semi-analytical method related to the effects of a weak shock wave on a submerged evacuated cylindrical elastic structure is proposed. The circular shell/acoustic medium interaction problem has already been tackled in the frequency domain with a full elastic model by Pathak and Stepanishen [1]. The purely transient case has only been achieved with simplified thin shell models based on the Love-Kirchhoff hypotheses for the structural dynamics, see for instance Iakovlev [2] and the references therein. The resulting radiated pressure field displays some discrepancies related to the A₀/S₀ waves when compared to the experimental data obtained by Ahyi et al [3]. Since the thin shell models are known to be restricted to the low frequency domain (the wavelengths in the structure and in the fluid must be larger than the shell thickness), and a weak shock wave may contain some high frequency components, the discrepancies should be overcome by the use of a full elastic model. This is done in this paper in a two-dimensional framework. The approach is based on the methods of Laplace transform in time, Fourier series expansions and separation of variables in space. It is shown that the previously reported drawbacks related to the A₀/S₀ waves are eliminated and the new approach results in much more realistic images of the radiated acoustic field when compared to experiments.

2. METHOD

We consider a two-dimensional elastic shell of constant thickness h, external radius Rₑ, density ρₑ and longitudinal and transversal wave velocities c_l and c_t, respectively. It is submerged in an infinite fluid medium of density ρ_f and subjected to a weak incident acoustic excitation. A schematic of the problem is shown in Figure 1. In the following, the variables are written in a dimensionless form: the lengths are normalized by Rₑ, the time by Rₑ/𝑐_l with c_l being the sound speed in the fluid, and the pressure by ρ_f/𝑐_l².

The displacements U into the shell are expressed in the two-dimensional theory of elasticity by the scalar potentials φ and ψ such that U = ∇φ + ∇×(ψe₃). The displacement potentials satisfy the dimensionless wave equations in the Laplace domain: ∇²φ - s²Ωₑ²φ = 0 and ∇²ψ - s²Ωₑ²ψ = 0, with s the Laplace variable and Ωₑ and Ωₖ respectively given by Ωₑ = c_l/𝑐_t and Ωₖ = c_l/𝑐_t. Performing the Fourier series expansions φ(r, θ, s) = ∑ₙ=₀ φₙ(r, s)cos(nθ) and ψ(r, θ, s) = ∑ₙ=₁ ψₙ(r, s)sin(nθ), and using the method of separation of variables, the potentials components are given by φₙ(r, s) = Aₙ(s)Jₙ(𝑖Ωₑsᵈ) + Bₙ(s)Yₙ(𝑖Ωₛሽ) and ψₙ(r, s) = Cₙ(s)Jₙ(𝑖Ωₛᵈ) + Dₙ(s)Yₙ(𝑖Ωₛᵈ), where Jₙ and Yₙ denote the classical Bessel functions. The potential coefficients Aₙ, Bₙ, Cₙ and Dₙ have to be determined with the boundary conditions: 𝜕φₙ(𝑟₁, 𝜃) = −pᵣ, 𝜕ψₙ(𝑟₁, 𝜃) = 0 and 𝜕ψₙ(𝑟₁, 𝜃) = 0. Here 𝑟₁ denote the inner dimensionless shell radius and p the fluid pressure. By virtue of the acoustic problem linearity, it is classically divided into three components: the incident pressure pᵢ which is a given data, the diffracted pressure p_d in order to balance the normal component of the incident wave velocity on the body surface, vₑ × e₃, and the last component pᵦ, representing the pressure radiated by the deformations of the submerged body. Using the classical elasticity relations and Fourier series expansions, the boundary conditions are expressed as functions of the potential coefficients, which yields a linear algebraic system of size 4 x 4 for each Fourier mode, very similar to that obtained by Pathak and Stepanishen (1994).
for the harmonic problem. The solution is straightforward: it consists of simple matrix inversions for each point of the Laplace variable and Fourier mode, numerical inversions of the Laplace transforms [4] and Fourier series summations. Once the shell displacements are obtained, the pressure in the fluid domain is derived employing the analytical response functions available for circular geometries, see for instance Iakovlev [2].

3. RESULTS

![Fig. 2. Pressure field in the fluid domain in MPa, obtained with the elastic model for the shell dynamics described in Section 2.](image)

![Fig. 3. Pressure field in the fluid domain in MPa, obtained with a thin shell model based on the Love-Kirchhoff hypotheses as in Iakovlev (2008).](image)

The resulting pressure field in the fluid domain is illustrated in Figure 2 for a shell of thickness $h/R = 0.06$ (with material parameters $\rho = 7800$ kg/m$^3$, $c_t = 5800$ m/s, $c_l = 3100$ m/s, $\rho_f = 1000$ kg/m$^3$ and $c_f = 1470$ m/s). The pressure field derived with a thin shell model is shown in Figure 3. The incident (I) and diffracted (D) waves are obviously the same since they are independent of the shell dynamics. Some discrepancies arise for the shell-induced waves, i.e. the pseudo-Rayleigh wave ($A_0$) and the Lamb wave ($S_0$). With the elastic shell model, these two waves are well distinguished and the agreement with experimental data provided by Ahyi et al [3] seems to be excellent (see figure 6(a) of that paper). As for the pressure field obtained with the thin shell model, the streaky pattern induced by the symmetric $A_0$ can be recognized just ahead the incident wave, but the phase velocity of its high frequency component is overestimated. The $S_0$ and $A_0$ waves cannot in this case be identified.

REFERENCES


ACKNOWLEDGMENTS

The first author gratefully acknowledges the financial support provided by the company DCNS Propulsion when he was a PhD student. The third author acknowledges financial support of the Killam Trusts and the Natural Sciences and Engineering Research Council of Canada.