A SEMI-ANALYTICAL APPROACH TO THE STUDY OF THE TRANSIENT ACOUSTIC RESPONSE OF CYLINDRICAL SHELLS

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1.   INTRODUCTION

A semi-analytical method related to the effects of a weak shock wave on a submerged evacuated cylindrical elastic structure is proposed. The circular shell/acoustic medium interaction problem has already been tackled in the frequency domain with a full elastic model by Pathak and Stepanishen [1]. The purely transient case has only been achieved with simplified thin shell models based on the Love-Kirchhoff hypotheses for the structural dynamics, see for instance Iakovlev [2] and the references therein. The resulting radiated pressure field displays some discrepancies related to the A_0/S_0 waves when compared to the experimental data obtained by Ahyi et al [3]. Since the thin shell models are known to be restricted to the low frequency domain (the wavelengths in the structure and in the fluid must be larger than the shell thickness), and a weak shock wave may contain some high frequency components, the discrepancies should be overcome by the use of a full elastic model. This is done in this paper in a two-dimensional framework. The approach is based on the methods of Laplace transform in time, Fourier series expansions and separation of variables in space. It is shown that the previously reported drawbacks related to the A_0/S_0 waves are eliminated and the new approach results in much more realistic images of the radiated acoustic field when compared to experiments.

2.   METHOD

We consider a two-dimensional elastic shell of constant thickness h, external radius R_s, density ρ_s and longitudinal and transversal wave velocities c_l and c_t, respectively. It is submerged in an infinite fluid medium of density ρ_f and subjected to a weak incident acoustic excitation. A schematic of the problem is shown in Figure 1. In the following, the variables are written in a dimensionless form: the lengths are normalized by R_s, the time by R_s/c_f with c_f being the sound speed in the fluid, and the pressure by $\rho_f c_f^2$.
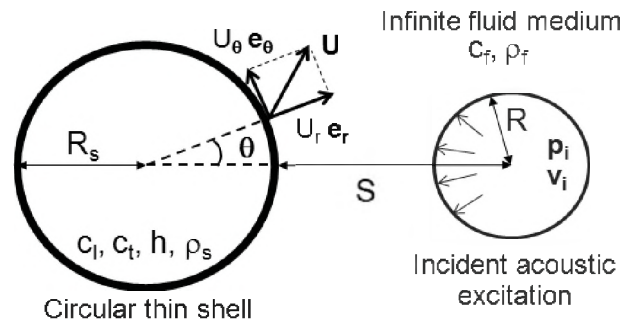

Fig. 1. Submerged two-dimensional circular cylindrical shell subjected to an incident acoustic excitation.

The displacements **U** into the shell are expressed in the two-dimensional theory of elasticity by the scalar potentials ϕ and ψ such that $\mathbf{U} = \nabla\phi + \nabla \times (\psi\mathbf{e}_z)$. The displacement potentials satisfy the dimensionless wave equations in the Laplace domain: $\nabla^2\hat{\phi} - s^2\Omega_l^2\hat{\phi} = 0$ and $\nabla^2\hat{\psi} - s^2\Omega_t^2\hat{\psi} = 0$, with s the Laplace variable and Ω_l and Ω_t respectively given by $\Omega_l = c_f/c_l$ and $\Omega_t = c_f/c_t$. Performing the Fourier series expansions $\hat{\phi}(r,\theta,s) = \sum_{m=0}^{\infty} \hat{\phi}_n(r,s)\cos(n\theta)$ and $\hat{\psi}(r,\theta,s) = \sum_{m=1}^{\infty} \hat{\psi}_n(r,s)\sin(n\theta)$, and using the method of separation of variables, the potentials components are given by $\hat{\phi}_n(r,s) = A_n(s) J_n(i\Omega_l sr) + B_n(s) Y_n(i\Omega_l sr)$ and $\hat{\psi}_n(r,s) = C_n(s) J_n(i\Omega_t sr) + D_n(s) Y_n(i\Omega_t sr)$, where J_n and Y_n denote the classical Bessel functions. The potential coefficients A_n, B_n, C_n and D_n have to be determined with the boundary conditions: $\hat{\sigma}_{rr}(1,\theta,s) = -\hat{p}|_{r=1}$, $\hat{\sigma}_{r\theta}(1,\theta,s) = 0$, $\hat{\sigma}_{rr}(r_i,\theta,s) = 0$ and $\hat{\sigma}_{r\theta}(r_i,\theta,s) = 0$. Here r_i denote the inner dimensionless shell radius and p the fluid pressure. By virtue of the acoustic problem linearity, it is classically divided into three components: the incident pressure p_i which is a given data, the diffracted pressure p_d in order to balance the normal component of the incident wave velocity on the body surface, $\mathbf{v_i} \cdot \mathbf{e_r}$, and the last component p_r, representing the pressure radiated by the deformations of the submerged body. Using the classical elasticity relations and Fourier series expansions, the boundary conditions are expressed as functions of the potential coefficients, which yields a linear algebraic system of size 4×4 for each Fourier mode, very similar to that obtained by Pathak and Stepanishen (1994)

for the harmonic problem. The solution is straightforward: it consists of simple matrix inversions for each point of the Laplace variable and Fourier mode, numerical inversions of the Laplace transforms [4] and Fourier series summations. Once the shell displacements are obtained, the pressure in the fluid domain is derived employing the analytical response functions available for circular geometries, see for instance Iakovlev [2].
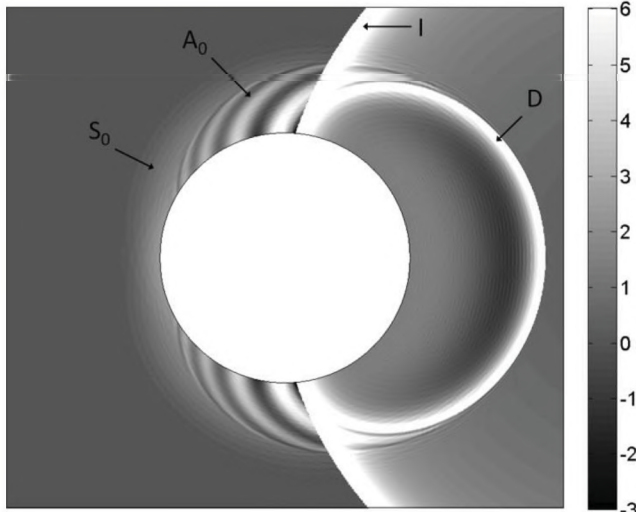
3. RESULTS

Fig. 2. Pressure field in the fluid domain in MPa, obtained with the elastic model for the shell dynamics described in Section 2.
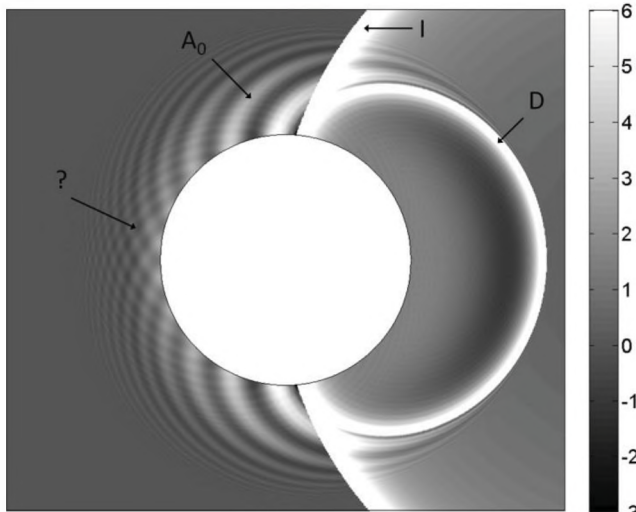

Fig. 3. Pressure field in the fluid domain in MPa, obtained with a thin shell model based on the Love-Kirchhoff hypotheses as in Iakovlev (2008).

The method is illustrated by computing the shell response to a weak underwater explosion. For this purpose, the similitude relation for the far-field shock-wave pressure profile provided by Price [5] is used. It consists of the hyper-acoustic pulse, $P(R,t) = P_c \left[\frac{a_c}{R}\right]^{1+A} F\left(\left[\frac{a_c}{R}\right]^B \frac{v_c}{a_c} t\right)$,

with F a known function given by $F(t) = 0.8251 e^{-1.338 t} + 0.1749 e^{-0.1805 t}$ valid for $t \le 7$, and R the distance from the explosive charge of radius a_c. The constants depend on the explosive and take the following values for the TNT: $P_c = 1.67\ GP_a$, $v_c = 1010\ m/s$, $A = 0.18$ and $B = 0.185$. As in Geers and Hunter [6], it is assumed that this pressure profile propagates in the far-field at the linear acoustic speed of sound and the incident velocity field is deduced through acoustic relations. Finally, the incident pressure and velocity fields required for the fluid-structure interaction problem, respectively p_i and $\mathbf{v_i}$, are expressed on the shell surface and derived analytically in the Laplace domain. Here, the incident fields are obtained by the detonation of 1 kg of TNT located at a dimensionless stand-off of $S = 2$.

The resulting pressure field in the fluid domain is illustrated in Figure 2 for a shell of thickness $h/R_s = 0.06$ (with material parameters $\rho_s = 7800$ kg/m^3, $c_l = 5800$ m/s, $c_t = 3100$ m/s, $\rho_f = 1000$ kg/m^3 and $c_f = 1470$ m/s). The pressure field derived with a thin shell model is shown in Figure 3. The incident (I) and diffracted (D) waves are obviously the same since they are independent of the shell dynamics. Some discrepancies arise for the shell-induced waves, i.e. the pseudo-Rayleigh wave (A_0) and the Lamb wave (S_0). With the elastic shell model, these two waves are well distinguished and the agreement with experimental data provided by Ahyi et al [3] seems to be excellent (see figure 6(a) of that paper). As for the pressure field obtained with the thin shell model, the streaky pattern induced by the symmetric A_0 can be recognized just ahead the incident wave, but the phase velocity of its high frequency component is overestimated. The S_0 and A_0 waves cannot in this case be identified.

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