BAYESIAN SOURCE TRACKING AND ENVIRONMENTAL INVERSION

Stan E. Dosso
School of Earth and Ocean Sciences, University of Victoria, Victoria BC Canada V8W 3P6, sdosso@uvic.ca

1. INTRODUCTION

This paper describes a Bayesian approach to two important and related inverse problems in underwater acoustics: localizing/tracking an acoustic source when ocean environmental properties are unknown, and determining environmental properties using acoustic data from an unknown (moving) source. The goal is not simply to estimate values for source and environmental parameters, but to determine parameter uncertainty distributions, quantifying the information content of the inversion. A common formulation is applied for both problems in which source parameters (location and spectrum) and environmental parameters are considered unknown random variables constrained by noisy acoustic data and by prior information on parameter values (e.g., physical limits for environmental properties) and on inter-parameter relationships (limits on horizontal and vertical source speed). Given the strong nonlinearity of the inverse problem, marginal posterior probability densities are computed numerically using efficient Markov-chain Monte Carlo importance sampling methods. Source tracking results are represented by joint marginal probability distributions over range and depth, integrated over unknown environmental parameters. The approach is illustrated with synthetic examples representing tracking a quiet submerged source and geoacoustic inversion using noise from an unknown ship-of-opportunity.

2. THEORY

Let $\mathbf{m}$ represent the model vector containing the unknown source locations and environmental parameters, and $\mathbf{d}$ represent the data vector containing measured acoustic fields, with the elements of both vectors considered random variables that obey Bayes rule, which may be written

$$P(\mathbf{m} \mid \mathbf{d}) \propto L(\mathbf{m}, \mathbf{d}) P(\mathbf{d}).$$

In the above equation, $P(\mathbf{m}|\mathbf{d})$ represents the PPD which quantifies the information content for the model parameters given both data information, represented by the likelihood function $L(\mathbf{m},\mathbf{d})$, and prior information $P(\mathbf{m})$. The likelihood can typically be written $L(\mathbf{m},\mathbf{d}) \propto \exp[-E(\mathbf{m},\mathbf{d})]$, where E represents the data misfit (log likelihood) function. The multi-dimensional PPD is typically characterized in terms of properties representing parameter estimates, uncertainties, and inter-relationships. Considered here are one- and two-dimensional marginal probability distributions, defined

$$P(m_i \mid \mathbf{d}) = \int \delta(m_i - m_i') P(\mathbf{m}' \mid \mathbf{d})\, d\mathbf{m}',$$
$$P(m_i, m_j \mid \mathbf{d}) = \int \delta(m_i - m_i') \delta(m_j - m_j') P(\mathbf{m}' \mid \mathbf{d})\, d\mathbf{m}'.$$

For nonlinear problems, such as acoustic localization and geoacoustic inversion, analytic solutions to the above integrals are not available, and numerical methods must be employed. Here integration is carried out using the method of fast Gibbs sampling (FGS), which applies Markov-chain Monte Carlo importance sampling methods in a principal-component parameter space [1, 2].

Let the complex acoustic pressure fields measured at an array of N sensors for F frequencies and S source positions be given by $\mathbf{d} = \{\mathbf{d}_{jk},\ j=1,S;\ k=1,F\}$. Assuming the data errors are complex, circularly-symmetric Gaussian-distributed random variables, the likelihood function is

$$L(\mathbf{m}, \mathbf{d}) \propto \prod_{j=1}^{S} \prod_{k=1}^{F} \exp\left\{ \mid \mathbf{d} - A_{jk} e^{i\theta_{jk}} \mathbf{d}_{jk}(\mathbf{m}) \mid^2 / \sigma_{jk}^2 \right\}$$

where $\mathbf{d}_{jk}(\mathbf{m})$ is the modelled acoustic pressure, A and θ represent the unknown source spectrum (amplitude and phase) and σ is the standard deviation Maximizing the likelihood (analytically) with respect to A , θ and σ leads to misfit function

$$E(\mathbf{m}, \mathbf{d}) = N \sum_{j=1}^{S} \sum_{k=1}^{F} \log_e \left\{ \mathbf{d}_{jk}^T \mathbf{d}_{jk} - \frac{\mid \mathbf{d}_{jk}^T \mathbf{d}_{jk}(\mathbf{m}) \mid^2}{\mid \mathbf{d}_{jk}^T(\mathbf{m}) \mathbf{d}_{jk}(\mathbf{m}) \mid} \right\}$$

3. EXAMPLES

The first example illustrates tracking a quiet submerged source in shallow water with little knowledge of environmental parameters. The environment and source parameters are illustrated in Fig. 1. Seabed geoacoustic parameters include the thickness h of an upper sediment layer with sound speed c_s, density ρ_s, and attenuation α_s, overlying a semi-infinite basement with sound speed c_b, density ρ_b, and attenuation α_b. The water depth is D, and the water-column sound-speed profile is represented by four parameters c_1–c_4 at depths of 0, 10, 50, and D m. Wide uniform prior distributions (search intervals) are assumed for all parameters. Acoustic data are measured at 300 Hz at

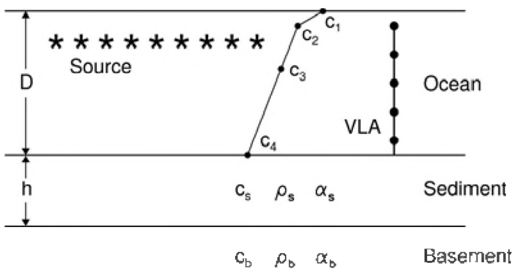

Fig. 1. Experiment geometry and model parameters.

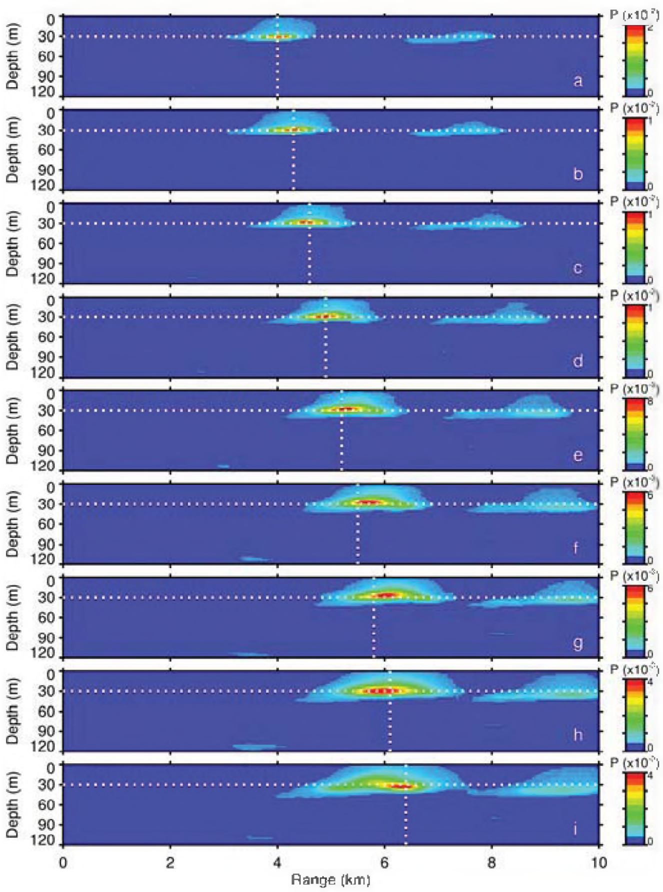

Fig. 2. PASs for the source-tracking example. Dotted lines indicate the true source depth and range

a vertical array consisting of 24 sensors at 4-m spacing from 26- to 118-m depth (simulated acoustic fields are computed using a normal-mode model). The track consists of an acoustic source at 30-m depth moving away from the array at a constant radial velocity of 5 m/s (~10 kts). Acoustic data are collected at the array once per minute for 9 minutes, corresponding to source-receiver ranges of 4.0, 4.3,…, 6.4 km. Random complex-Gaussian errors are added to the data to achieve a signal-to-noise ratio (SNR) that varies from –2 to –8 dB with increasing range along the track. Figure 2 shows probability ambiguity surfaces (PASs), which consist of joint marginal probability distributions for source range and depth integrated over all unknown environmental parameters via FGS, with the additional constraint (prior information) of a maximum source velocity of 10 m/s in the radial and 0.06 m/s in the vertical. The PASs have a strong maximum near the true source location, although weaker secondary maxima are also evident.

The second example simulates geoacoustic inversion using noise lines emanating from a moving ship-of-opportunity of unknown location. The environmental parameters, sensor array, and source track are identical to the tracking example, except that the source depth is 6 m with prior bounds of 2-10 m (i.e., prior knowledge that the source is a surface ship); source range is considered unknown over 0-10 km. Acoustic data are considered at two frequencies of 300 and 350 Hz, with SNR of 4 dB at the shortest source range decreasing with range to approximately –3 dB. Figure 3 shows marginal probability distributions computed for the environmental parameters, indicating a good resolution of the seabed sound-speed structure (h, c_s, c_b) despite the lack of knowledge of source location and water-column sound-speed profile.

REFERENCES

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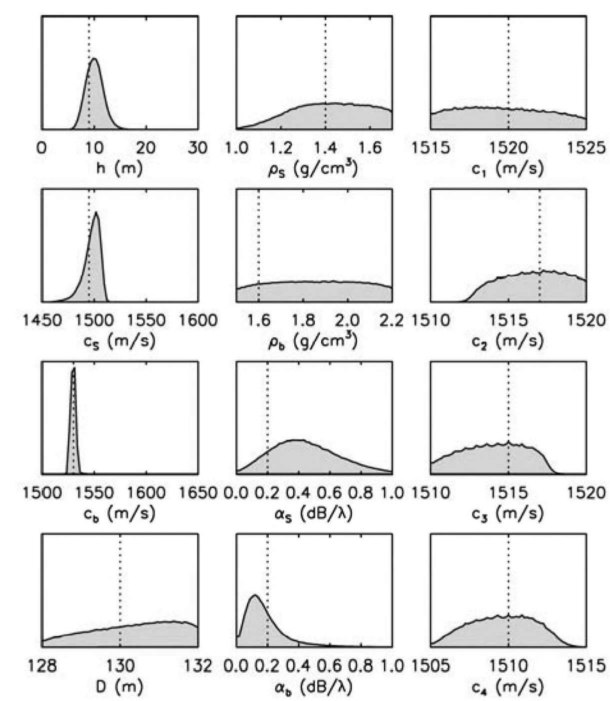

Fig.3. Posterior marginal probability distributions for the ship-noise geoacoustic inversion example.