TWO APPROACHES TO SOURCE TRACKING IN AN UNKNOWN OCEAN ENVIRONMENT

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1. INTRODUCTION

This paper compares two approaches, within a Bayesian context, to localizing and tracking a low-level acoustic source in the ocean when environment properties are poorly known. Optimization is based on determining the source and environmental parameters that maximize the multi-dimensional posterior probability distribution (PPD). Marginalization integrates the PPD over nuisance environmental parameters to obtain marginal probability distributions over source range and depth, and the optimal track is determined from these marginal distributions. The question addressed here is which method yields track estimates that are, on average, closer to the true track.

2. THEORY

Let **m** and **d** represent the model and data vectors, respectively, with elements considered random variables that obey Bayes rule, which may be written

 $P(\mathbf{m} \mid \mathbf{d}) \propto L(\mathbf{m}, \mathbf{d})P(\mathbf{d}).$

In the above equation, $P(\mathbf{m}|\mathbf{d})$ represents the PPD which quantifies the information content for the model parameters given both data information, represented by the likelihood function $L(\mathbf{m}, \mathbf{d})$, and prior information $P(\mathbf{m})$. The likelihood can typically be written $L(\mathbf{m}, \mathbf{d}) \propto \exp[-E(\mathbf{m}, \mathbf{d})]$ where E represents an appropriate data misfit function.

The multi-dimensional PPD is typically characterized in terms of parameter estimates and uncertainties, such as the maximum *a posteriori* (MAP) model and marginal probability distributions, defined by

$$\hat{\mathbf{m}} = \operatorname{Arg}_{\max} \{ P(\mathbf{m} \mid \mathbf{d}) \}$$

$$P(m_i, m_i \mid \mathbf{d}) = \int \delta(m_i - m'_i) \,\delta(m_i - m'_i) P(\mathbf{m}' \mid \mathbf{d}) \, d\mathbf{m}'.$$

Optimization seeks the source track and environmental parameters that minimize the misfit to acoustic data (i. e., the model MAP estimate) For efficiency, the optimization is carried out only over environmental parameters, since the most probable source track (subject to source velocity constraints) given the environmental parameters can be calculated using the Viterbi algorithm [1]. Optimization is a generalization of the focalization technique [2] to source tracking. Both adaptive simplex simulated annealing [3] and differential evolution have been used for optimization with good success.

Marginalization requires integration over the environmental parameters [4] to obtain track marginal distributions. Here integration is carried out using the method of fast Gibbs sampling (FGS), which applies Markov-chain Monte Carlo importance sampling methods in a principal-component parameter space. The Viterbi algorithm can then be applied to determine the optimal (most probable) track from the marginal distributions.

3. **RESULTS**

The optimization and marginalization approaches are compared here with a synthetic example illustrated in Fig. 1. The geoacoustic parameters include the thickness hof an upper sediment layer with sound speed c_s , density ρ_s , and attenuation a_s , overlying a semi-infinite basement with sound speed c_b , density ρ_b and attenuation α_b . The watercolumn sound speed profile is represented by four unknown sound speeds c_1 - c_4 at depths of 0, 10, 50, and D m, where D is the water depth. A low-level 300 Hz source travels at constant depth of 20 m and at a constant velocity of 5 m/s. Acoustic fields from this source are recorded at a 24-sensor vertical array (VLA) once every minute for nine minutes. during which the source moves from 4-km to 6.4-km range. For the inversions, wide prior bounds are applied for the geoacoustic and water column parameters, and the source horizontal and vertical velocities are constrained to be less than 10 m/s and 0.07 m/s, respectively.

For the study carried out here, acoustic data are considered at six different signal-to-noise ratios (SNRs), with average SNRs along the track ranging from -11 to -3 dB (the SNR decreases along the track by approximately 6 dB due to the increasing range). At each SNR, 20 different noisy data sets were inverted using both optimization and marginalization. Figure 2 shows the probability of an acceptable track (PAT), defined as mean absolute depth and range errors less than 10 m and 500 m, respectively, for



Fig. 1. Geometry and model parameters.

optimization and marginalization, including one standarddeviation error bars. Also included in the figure for reference are PATs computed via the Viterbi algorithm using either exact knowledge of the environmental parameters or environmental parameters drawn at random from the prior bounds. The key result is that marginalization gives significantly better average results than optimization, particularly for SNRs of -9 to -3 dB. Marginalization has the added benefit that the marginal distributions used to calculate the optimal track provide a measure of track uncertainty. Figure 3 shows an example at SNR = -6 dB with one mean-deviation uncertainties about the optimal track.

4. SUMMARY

The study carried out here indicates than marginalization significantly outperformed optimization for source tracking in an unknown ocean environment. In addition, marginalization also provides a measure of the track uncertainty. However, the integrations required in marginalization require greater computational time effort than optimization.

REFERENCES

[1] Viterbi, A. J. (1967) Error bounds for convolutional codes and an asymptotically optimum decoding algorithm. IEEE Trans. Inf. Theory IT-13, 268-269.

[2] Collins, M. D., and W. A. Kuperman (1991). Focalization: Environmental focusing and source localization. J. Acoust. Soc. Am., vol 90, 1410-1422.

[3] Dosso, S. E., M. J. Wilmut and A. L. Lapinski (2001). An adaptive hybrid algorithm for geoacoustic inversion. IEEE. J. Ocean. Eng., vol 26, 324-336.

[4] Dosso, S. E. and P. L. Nielsen (2002). Quantifying uncertainty in geoacoustic inversion II: A fast Gibbs sampler approach. J. Acoust. Soc. Am., vol 111, 143-159.



Fig. 2. Comparison of optimization (open circles) and marginalization (filled circles) results for tracking. Dotted and dashed lines indicate results using exact and random environmental parameters, respectively.



Fig. 3. Example of track estimate and uncertainty. The true track is indicated by open circles and the estimated track by filled circles with mean-deviation uncertainties.