

A PERFECTLY MATCHED LAYER TECHNIQUE FOR LATTICE BOLTZMANN METHOD

Alireza Najafi-Yazdi, Luc Mongeau

Dept. of Mech. Eng., McGill University, 817 Sherbrooke St. West, Montreal, QC, Canada, H3A 2K6

1. INTRODUCTION

In recent years, the Lattice Boltzmann Method (LBM) has emerged as a promising computational technique in fluid dynamics. LBM has some intrinsic advantages over conventional Navier-Stokes schemes [1] such as ease of parallel implementation.

The main difference between LBM and conventional Navier-Stokes schemes is that, Navier-Stokes equations are derived explicitly for the macroscopic properties of the fluid, while LBM's involve the solution of lattice-Boltzmann equation (LBE) by explicitly tracking the development of particle distribution functions either at the mesoscopic or the microscopic scale. Using the Chapman-Enskog expansion, the compressible Navier-Stokes equations can be recovered from the LBE at the hydrodynamic limit.

Recently, the LBM has been evaluated and utilized for some aeroacoustics applications. However, robust nonreflective boundary conditions are still needed for LBM. As underlined in one recent study [2], little work has been reported on this topic.

In the present study, a boundary condition was developed based on the perfectly matched layer (PML) concept introduced by Berenger for numerical simulations of electro-magnetic fields [22]. The most significant feature of the PML technique is the fact that it creates absorbing layers that are theoretically non-reflective for any angle and frequency of incident wave. Moreover, the intrinsic linearity and computational scheme robustness of LBE prevent instabilities and complexities associated with nonlinear convection terms which are present in Euler and Navier-Stokes equations.

2. A PML FORMULATION FOR LATTICE BOLTZMANN METHODS

The lattice Boltzmann equation is one discrete form of the continuous Boltzmann equation:

$$\frac{\partial f}{\partial t} + (\vec{\xi} \cdot \vec{\nabla}) f = \Omega, \quad (1)$$

where Ω is the inter-molecular collision operator. In order to facilitate solution of the Boltzmann equation, the collision operator is usually simplified using the Bhatnagar-Gross-Krook (BGK) approximation:

$$\Omega = -\frac{f-f_{eq}}{\tau}, \quad (2)$$

where τ is the relaxation time and f_{eq} is the local equilibrium Maxwell-Boltzmann distribution. The hydrodynamics properties such as density, momentum, kinetic energy, and others can be obtained by different moments of the equilibrium distribution function in the phase space. To enable numerical integration of these moments, the distribution function is obtained only for certain velocity directions which are the abscissas of a Gaussian-type quadrature. These velocity directions form a $DnQm$ lattice, where n is the number of dimensions of the flow field and m is the number of velocity directions within the lattice. A D2Q9 lattice which is commonly used in 2D simulations is shown in Figure 1.

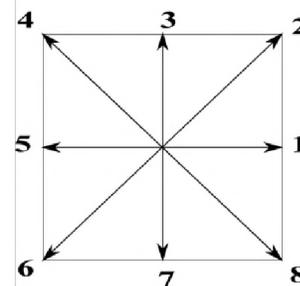


Figure 1. A D2Q9 lattice used in 2D simulations.

The decomposition of the equilibrium distribution function into the sum of a mean component, which corresponds to the hydrodynamic field, and a perturbation component, which corresponds to the acoustic perturbation, yields a set of equations consistent with the Boltzmann equation at the interface between the absorbing zone and the interior domain (see Figure 2).

The following formulation is proposed for the absorbing zone in a lattice Boltzmann simulation:

$$\frac{\partial f}{\partial t} + (\vec{\xi} \cdot \vec{\nabla}) f = \Omega - \Omega^{\text{PML}}, \quad (3)$$

where

$$\Omega^{\text{PML}} = \sigma (\vec{\xi} \cdot \vec{\nabla}) Q + 2 \sigma (f_{eq} - \overline{f_{eq}}) + \sigma^2 Q,$$

and

$$\frac{\partial Q}{\partial t} = f_{eq} - \overline{f_{eq}}$$

The details of the derivation are presented in Ref [4]. It is notable that the PML role is encapsulated in only one single additional term to the collision operator. The damping coefficient σ controls the decay rate of the waves entering the PML zone. Eq. (3) can then be made discrete in the same manner as the classical lattice Boltzmann method. The damping coefficient is predefined by the user at the beginning of the simulation considering the thickness of the PML region.

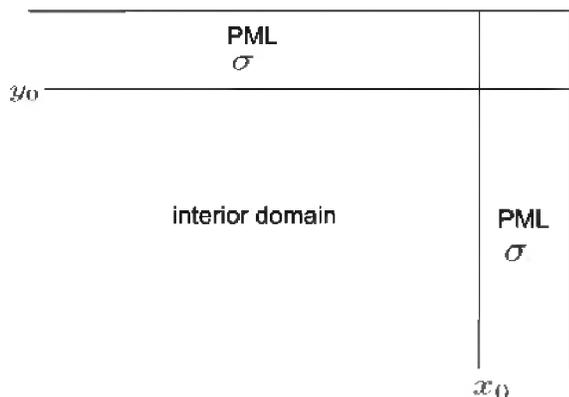


Figure 2. PML setup in a 2D simulation.

4. Numerical Examples and Discussion

4.1. Propagation of a Gaussian Acoustic Pulse

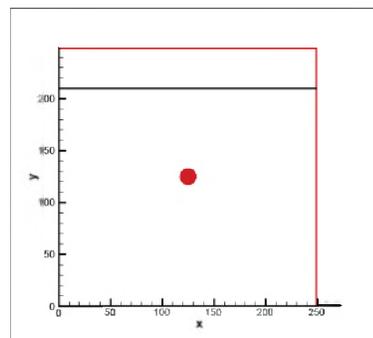
One classical problem to assess the performance of numerical boundary conditions is the propagation of a Gaussian pulse. The following initial conditions were imposed in this case:

$$\rho = 1 + 0.0001 \exp\left(-\ln 2 \frac{x^2 + y^2}{9}\right)$$

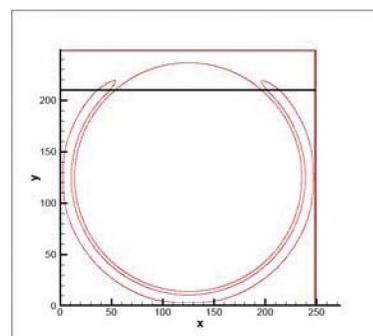
$$u = 0$$

$$v = 0$$

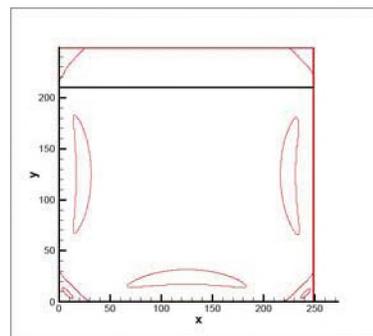
All quantities are made non-dimensional using the grid spacing and mean density. The pulse was initially located at the center of a 256 by 256 nodes grid. A 40-lattice wide PML was created between the interior domain and the northern boundary while conventional outlet (zero normal gradients) BCs were chosen for all other boundaries. A damping coefficient of 0.03 was chosen for the PML. The attenuation of the wave in the PML is demonstrated in Fig 3. Almost no reflection from the northern boundary was observed. Similar simulations have also been performed with PML boundary conditions on all boundaries, and in situations where the Gaussian pulse exits the boundary in presence of a mean flow with arbitrary direction. Excellent results were obtained for all cases.



(a)



(b)



(c)

Figure 3. Gaussian pulse propagation; (a) $t = 0$; (b) $t = 200$; (c) $t = 280$; The PML is located on the north boundary. All dimensions are normalized by lattice units.

REFERENCES

- [1] D. Yu, R. Mei, L.S. Luo, and W. Shyy. Viscous flow computations with the method of lattice Boltzmann equation. *Progress in Aerospace Sciences*, 39(5):329–367, 2003.
- [2] EWS Kam, RMC So, and RCK Leung. Lattice Boltzmann Method Simulation of Aeroacoustics and Nonreflecting Boundary Conditions. *AIAA Journal*, 45(7):1703–1712, 2007.
- [3] J. Berenger. PA perfectly matched layer for the Absorption of electromagnetic waves. *Journal of Computational Physics*, 114(2):185–200, 1994.
- [4] A. Najafi-Yazdi, L. Mongeau, “A Perfectly Matched Layer Technique for Lattice Boltzmann Equations,” to be submitted to the *Journal of Computational Physics*.