LOW-FREQUENCY ROOM DEMERIT ANALYSIS AND EQUALIZATION USING PROPERLY-MODELED SOURCE TERMS IN FINITE-DIFFERENCE TIME-DOMAIN SIMULATIONS

Ryan J. Matheson
[1]Dept. of Physics & Astronomy, University of Waterloo, 200 University Ave. West., Ontario, Canada, N2L 3G1
ryan.j.matheson@gmail.com

1. FDTD Source Term Derivation

In deriving the Finite Difference Time Domain Method (or FDTD Method) for Acoustics we require two essential equations. The first is the Linear Inviscid Force Equation, or Newtons' Equation,

$$\nabla p = -\rho \frac{\partial}{\partial t} \boldsymbol{u} \qquad (1)$$

The second is the Equation of Continuity,

$$\frac{1}{\rho c^2} \frac{\partial}{\partial t} p = \frac{q(r,t)}{\rho} - \nabla \cdot \boldsymbol{u} \qquad (2)$$

where p is the sound pressure, **u** is the particle velocity vector, ρ is the density of the medium and q(r,t) is the function that defines the rate of creation of fluid. Normally this equation is quoted without the function q, but if one carefully follows the FDTD derivation method laid out in [1] then it can be shown that at a point the equation for pressure in the FDTD scheme can be given by,

$$\begin{aligned} p(x,T) &= p(x,T-k) + k \cdot c^2 \cdot q\left(x, T-\frac{k}{2}\right) \\ &- \frac{c^2 \cdot \rho \cdot k}{h}\left[u_x\left(x+\frac{h}{2}, T-\frac{k}{2}\right) - u_x\left(x-\frac{h}{2}, T-\frac{k}{2}\right)\right]. \qquad (3) \end{aligned}$$

Equation (3)[1] contains the function q which is the rate of creation of fluid in the system, which has units of $kg \cdot m^{-3} \cdot s^{-1}$. But we want a relation that contains the volume velocity function Q, which has units of $m^3 \cdot s^{-1}$. Therefore we can relate q and Q by multiplying q by a volume and dividing it by the density of the medium. Thus

$$Q = \frac{\Delta V}{\rho} \cdot q \quad \text{or} \quad q = \frac{\rho}{\Delta V} \cdot Q \qquad (4)$$

Here $\Delta V = \Delta x \cdot \Delta y \cdot \Delta z = h^3$. Substituting (4) into (3) yields,

$$\begin{aligned} p(x,T) &= p(x,T-k) + \frac{k \cdot c^2 \cdot \rho}{\triangle V} Q\left(x, T-\frac{k}{2}\right) \\ &- \frac{c^2 \cdot \rho \cdot k}{h}\left[u_x\left(x+\frac{h}{2}, T-\frac{k}{2}\right) - u_x\left(x-\frac{h}{2}, T-\frac{k}{2}\right)\right]. \qquad (5) \end{aligned}$$

We require the use of Q in this equation because conventional cone loudspeakers produce volume velocity, and at low frequencies are approximately point sources. This is exactly what equation (5) is, a pressure point source that is driven by a volume velocity.

It should be noted that the pressure at a distance r away from a volume velocity point source is given by

$$p(r) = \frac{\rho \cdot A(t)}{4\pi \cdot r},$$

where A(t) is the volume acceleration, or dQ(t)/dt. This result is observed in the simulations.

[1] Equations (3) and (5) are the 1-D case.

2. Cancellation Methods

2.1 End-Wall Speaker Placement

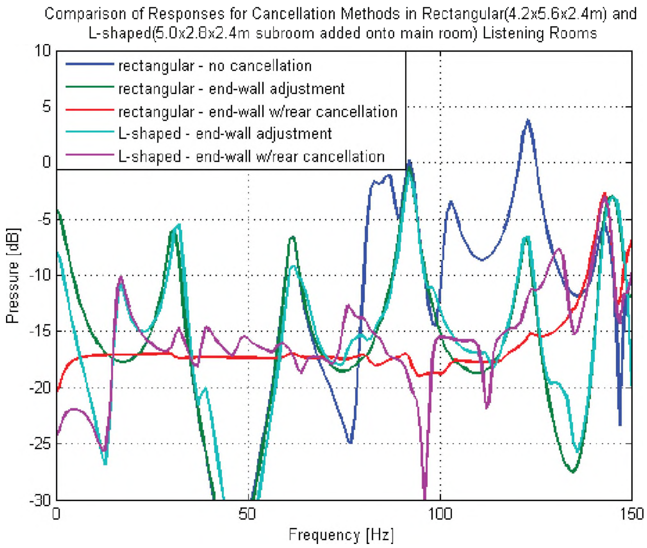
One of the easiest ways to equalize low frequencies in a room is to adjust the speaker placement such that fundamental modes of vibration in the room are not excited. For example, the corner of a room is a good way to excite all modes of vibration in a room. A possible placement scheme for a stereo setup in a room with dimensions width(W) x length(L) x height(H) would be to place the two speakers at (W/4, L, H/2) and (3W/4, L, H/2). At very low frequencies, in reasonable sized listening rooms in which L is the largest dimension, this speaker placement has the effect of creating a plane wave along the length of the room[2].

2.2 Rear Cancellation

An active method of low frequency room equalization is the rear cancellation method. If we take our example low-frequency speaker placement in the last sub-section, (W/4, L, H/2) and (3W/4, L, H/2), then what we would want to do is place an identical pair of speakers at the opposite end of the room at (W/4, 0, H/2) and (3W/4, 0, H/2). These rear low-frequency speakers would then be fed an inverted and delayed version of the signal that would be sent to the front speakers.

2.3 Cancellation Results

As can be observed from the figure below, we see that the end-wall adjustment helps equalize things a bit because we are no longer exciting the lowest modes that correspond to the W dimension. This does not affect the lowest frequency modes that are attributed to the L dimension though, which causes the largest low frequency resonances. The L-dimensional modes are taken care of by the rear cancellation method rather well as we observe that we have removed all of the modes below roughly 130Hz; where the troublesome low frequency resonances occur.



2.4 Cancellation of L-shaped rooms

Simulating and equalizing rectangular listening rooms is relatively easy but most rooms are not perfectly closed-off rectangles. Common rooms have doorways, windows or hallways. Outside doorways and windows are openings that open up to the larger outside world so these will not add any additional resonance modes to the room. On the other hand, closed-off hallways add an extra dimension to the room that creates coupled modes of vibration that, typically, adds an additional low frequency resonance mode. This additional mode can be seen in the above figure as the lowest mode for the L-shaped room plots, and it is not affected by our cancellation techniques.

3. Room Demerit

By visually comparing the room responses in the figure it is easy to tell which responses are more desirable. But all we can say from the plot is that the curves with cancellation look better than the ones without because they look smoother. This is a qualitative measure. But we all know from high school science that quantitative measures are more defensible. Thus we present two possible ways to calculate a 'Room Demerit' value which are variations of the demerit found in [3].

$$D1 = \frac{1}{N}\sum_{n=1}^{N}\left[\left|\frac{P_{unsmoothed}[n] - P_{smoothed}[n]}{P_{smoothed}[n]}\right|\right] \quad (6)$$

$$D2 = \frac{1}{N}\sum_{n=1}^{N}\left[\frac{(P_{unsmoothed}[n] - P_{smoothed}[n])^2}{P_{smoothed}[n]^2}\right] \quad (7)$$

In these calculations each mic uses its own smoothed result. Another approach might be to use the same smoothed result for all mics.

In most cases where smoothing is involved people like to use octave smoothing. This is done because at low frequencies it can be argued, based on how people hear in the bass region, that a constant bandwidth smoothing is more appropriate, a bandwidth of 60Hz over the range from 0 to 100Hz was chosen that produced the results in the following table.

Table of Results from Room Demerit Calculation of 25 mics.

	rectangular – no cancellation	rectangular – end-wall adjustment	rectangular – w/rear cancel	L-shaped-end-wall	L-shaped-w/rear cancel
Mean(D1)	1.0437	1.1750	.0772	1.0314	.5610
Mean(D2)	2.5454	3.7271	.0133	2.5206	.8731
STD(D1)	.1060	.0495	.0106	.0465	.0427
STD(D2)	.9943	.5385	.0023	.4161	.1782
SDOM(D1)	.0106	.0049	.0011	.0046	.0043
SDOM(D2)	.0994	.0539	.0002	.0416	.0178

When performing any sort of room equalization technique it is best to compare results over several mic positions. History has shown that equalizing one area very easily un-equalize another. Thus the demerit data is collected over 25 microphones covering a $4m^2$ area in our simulated room, and the mean demerit value is computed from these mics. We also know that from statistics that when using the mean to assess a distribution it is also helpful to calculate the Standard Deviation(STD) and the Standard Deviation of the Mean(SDOM). The STD and SDOM provide a measure of how consistent the distribution is about the mean value and is of interest to us because when we equalize the room we not only want a flat response, but we also want to ensure that the response is consistent across the listening room, so that anyone sitting anywhere in our room will receive the same performance as everyone else, thus attempting to change our 'acoustic sweet spot' into an 'acoustically sweet area'.

REFERENCES

[1] S. Krarup Olesen, "Low Frequency Room Simulation using Finite Difference Equations", presented at the 102nd AES Convention, 1997 March 22-25, Munich, Germany. Paper 4422

[2] A. Celestinos and S. Birkedal Nielsen, "Low Frequency Sound Field Enhancement System for Rectangular Rooms using Multiple Low Frequency Loudspeakers", presented at the 120th AES Convention, 2006 May 20-23, Paris, France. Paper 6688

[3] J. Vanderkooy, "Multi-Source Room Equalization: Reducing Room Resonances", presented at the 123rd AES Convention, 2007 October 5-8, New York, NY, USA. Paper 7262