EIGENFREQUENCY ANALYSIS OF FLUID-FILLED PIPES

N M Alam Chowdhury[1,*], Zaiyi Liao[2], Lian Zhao[1] and Ramani Ramakrishnan[2]
[1]Dept. of Electrical & Computer Engineering, Ryerson University
[2]Dept of Architectural Science, Ryerson University,
[*]350 Victoria St., Toronto, ON, Canada, M5B 2K3, nmalamc@yahoo.com

1. INTRODUCTION

The eigenfrequency analysis of piping system is used in thermodynamic or chemical engineering process for many years, which are generally subjected to static or time dependent variable loads. In time dependent case, these loads generate acoustic sources which may lead to excessive noise level in the piping system. Many researches [1-4] were performed on this matter for its technological importance and the performance degradation of the piping system. In the prestressed concrete cylinder pipe (PCCP), the acoustic signals are generated through the breakage or sliding of reinforced wires. These signals feature certain frequency spectral characteristics that are close to the eigenfrequency of the piping system. This signal degrades the performance of the condition monitoring of piping system. As far as the authors' knowledge, there is no systematic theoretical analysis on this problem in the open literature. In this work, the eigenfrequency analysis of these events is taken as a principal interest.

The fundamental element of such an analysis is to investigate the eigenfrequency of the wave propagation impacted by the complexity of fluid medium surrounded by a large concrete pipe buried in the ground. The mathematical model is developed using Navier's equation of motion for acoustic wave in frequency domain. The interaction between the fluid and the surrounding layers are modeled based on Newton's law of motion and principle of virtual work. The results of the eigenfrequency analysis are presented and compared with available analytical solutions.

2. MODEL FORMULATION

The wire break or slip generated acoustic signal propagates through the pipe structure and the fluid in the pipe. Let us assume that, the time harmonic acoustic pressure (p) propagates uniformly through a lossless fluid. Therefore, using pressure-velocity relation in Navier's scalar velocity potential equation, the acoustic pressure equation in frequency domain, can be written as [5]

$$\nabla^2\left(-\frac{1}{\rho_F}\right)p-\frac{\omega^2 p}{\rho_F \mathrm{v}_F^2}=0, \qquad (1)$$

where ρ_F is the fluid density, v_F is the speed of the signal in the fluid medium and ω is the angular frequency.

According to Newton's law of motion in equilibrium, this external acoustic pressure forces generate the same internal body forces in the pipe structure. These causes undergo unrelated but consistent displacements and deformations. This displacement in the structure generates the same but opposite amount of traction forces on the pipe wall. The interactions between different layers are modeled using appropriate boundary conditions [5].

The eigenfrequency of the model can be obtain by solving the homogeneous pressure equation given in Eq.(1). The solution becomes zero except at a discrete set of eigenfrequencies with a well-defined shape of undefined magnitude.

3. NUMERICAL STUDY

Let us consider a uniform and smooth fluid-filled PCCP surrounded by the outer formation (e.g. soil) as shown in Fig.1. For simplicity, assume the pipe structure consist of high-strength concrete only and is filled with static fluid (water). Damping is absent in all medium. The basic properties of the medium are given in Table-1.

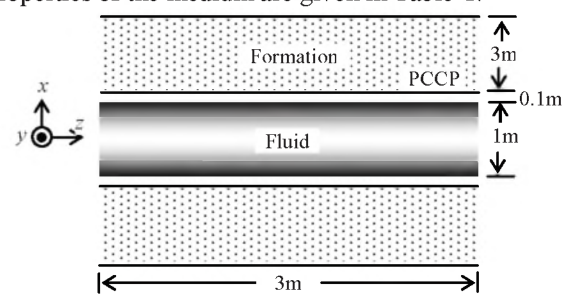

Figure 1. Typical model geometry with dimensions.

Table 1. Basic properties of the medium.

Properties	Water	Concret
Density (ρ), kg/m^3	997	2400
Acoustic wave Speed (v), m/s	1500	--
Elastic modulus (E), Pa	--	40^9
Poisson's ratio (γ)	--	0.33

4. RESULTS AND DISCUSSION

The model is simulated for the rigid and elastic pipe with infinite and finite stiffness, respectively and for the various dimensions of pipe and medium characteristics. For the rigid pipe the simulated results are compared with the analytical solutions [5]. Similar analysis is also performed for elastic pipes, where analytical solutions are unknown. However, the simulated results are analyzed and validated as per theoretical and physical concept.

In this paper, the simulated results of water-filled concrete pipe (elastic pipe with finite stiffness) with different radius (R) are presented (Fig.2), as an example. The acoustic signal propagating in the water has an interaction with the pipe structure, and vice versa. As a result, eigenfrequency, cut-off frequency and phase speed have variable quantities based on the pipe and medium characteristics.

Figure 2 shows the eigenfrequency analysis of elastic pipe with finite stiffness at different radii. From the figure it is seen that, up to the first cut-off point, the eigenfrequencies are separated at regular intervals. After that, it changes very rapidly. Moreover, increasing the radius reduces the cut-off frequency of pseudo-rayleigh modes. As far as authors' knowledge, there is no analytical solution in the open literature to compare these values. However, it has a similar trend of rigid pipe solution, which can be compared with the analytical solution [5]. This means that, in acoustic emission (AE) monitoring system, it is possible to collect more AE events for the larger diameter pipe by placing the sensors at far distance. On the other hand, this increased radius generates additional vibrating signal which produces complicated dispersion characteristics at the receiving end (sensors). Therefore, care should be taken to indicate the active distress in the pipeline.

At low frequency stoneley mode, increasing the inner radius reduces the pipe stiffness. Consequently, both pipe structure and fluid dissipate its energy due to very low viscosities and material damping, which reduces the phase speed of the propagating acoustic signal. The phase speed of the acoustic wave inside the pipe in the fluid medium can be obtained from the velocity of the first eigen wave. The simulated result is compared with the analytical tube wave [6] velocity (v_T), which is given below in the caption of respective graph, and found a good agreement. In AE monitoring system, this phase speed is important to localize the events.

Similarly, we can observe the effect of pipe thickness, elasticity and outer formation on the acoustic signal propagation using eigenfrequency analysis [5].

5. CONCLUSIONS

The impacts of the pipe radius on the stoneley and rayleigh modes, and the phase speeds of the system are studied. The results show that the radius has significant impacts on the rayleigh modes but not on the stoneley modes of propagation. Extra harmonic eigen waves are observed in the rayleigh modes. This is in controversy with the analytical solutions. These eigen wave solutions are the fundamentals for a number of more complex wave propagation studies, such as system vibration, pulse propagation and perturbation analysis.

Overall, the speed of the acoustic signals propagating through pipes is not constant and is affected by the pipe profiles and surrounding formations. The acoustic signals recorded by the sensors used in current industry may have been noised by the system resonance and thus the testing data should be analyzed more rigorously. The study presented in this paper produces the fundamentals on which better non-destructive pipe performance testing technologies can be designed.

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ACKNOWLEDGEMENTS

This work has been partially supported by the Ontario Centres of Excellence (OCE) under Grant No. EE50196 and Natural Sciences and Engineering Research Council (NSERC) of Canada under Grant No. DG 293237-04 and 313375-07.

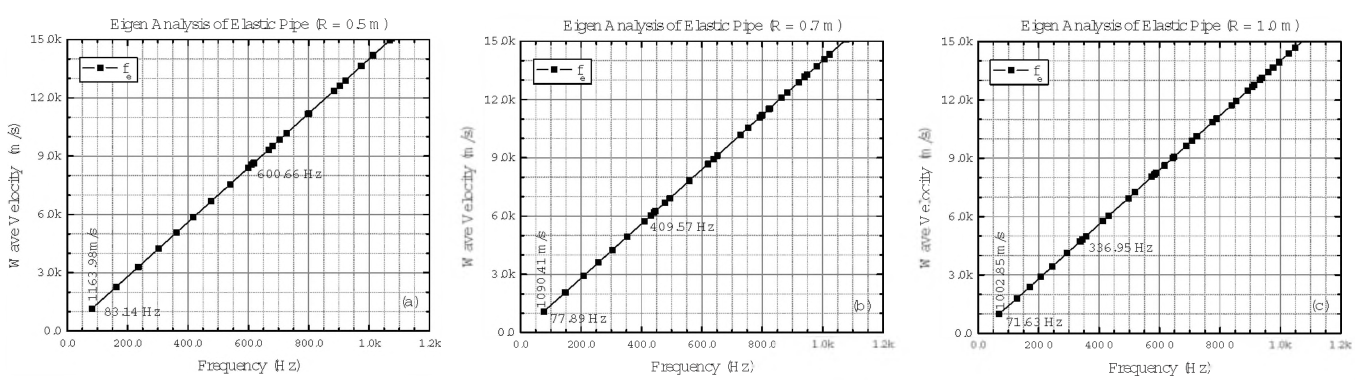

Figure 2. Eigenfrequency of water-filled elastic pipe - (a) R = 0.5 m (v_T = 1164.30 m/s), (b) R = 0.7 m (v_T = 1094.06 m/s), and (c) R = 1.0 m (v_T = 1007.35 m/s).