MODAL SOLUTIONS OF THE ACOUSTIC WAVE PROPAGATION THROUGH FLUID-FILLED PIPES

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1. INTRODUCTION

In the pipeline, the guided waves propagate with very complicated wave structures due to a mixture of multi-modes and mode conversion, which prevents guided waves from being widely used. Modal analysis and simulation of guided wave propagation are very useful for solving these problems. In the piping system, the scope of this analysis includes the evaluation of dispersion behavior of guided waves in hollow or liquid-filled cylinders. In long-range inspection of pipes, this guided waves largely reduces inspection time and costs compared to the ordinary point by point testing in large pipeline [1-3].

In the prestressed concrete cylinder pipe (PCCP), the acoustic waves are generated through the breakage or sliding of reinforced wires. These waves have different frequency spectra, which consist of low to high frequency signals, and propagate through the fluid column inside the pipe as a guided wave. At low frequencies, the wave propagates as an evanescent mode with plane wavefront. These waves cannot propagate for long distances, where both pipe structure and fluid dissipates its energy due to very low viscosities and material damping [4]. At high frequencies, it has also propagating mode which exhibits oscillatory amplitude in the fluid and a decaying amplitude in the pipe structure, and propagates a far distance. The number of propagating modes grows proportional to the signal frequency. Consequently, more modes cause complicated dispersion characteristics. In elastic pipe, such waves have strong dispersive phenomena [142]; as a result, there exist several wave modes in the same propagating signal. Therefore, the modal analysis is important to determine the dynamic characteristics of the system.

The mathematical model is developed based on Navier's equation of motion. The model is solved to obtain the modes of propagation and phase wave of acoustic wave in for various radii of the pipe. The finite-element analysis is used to simulate the model and the results are compared with the analytical solutions.

2. MATHEMATICAL MODEL

Let us assume that the wire break or slip generated acoustic emission (AE) wave propagates through a lossless fluid in the pipe. Therefore, using pressure-velocity relation in Navier's scalar velocity potential equation, the inhomogeneous acoustic pressure equation in frequency domain, can be written as [5]

$$\nabla^2\left(-\frac{1}{\rho_F}\right)p-\frac{\omega^2 p}{\rho_F v_F^2}=g, \qquad (1)$$

where ρ_F is the fluid density, v_F is the speed of the signal in the fluid medium, ω is the angular frequency and g is the AE source.

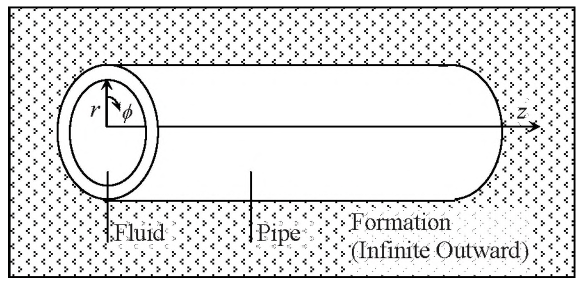

Figure 1. Schematic of fluid-filled PCCP.

2.1 Modal Analysis

Let us assume that the acoustic pressure varies harmonically in longitudinal direction (say z) as shown in Fig.1. Therefore, the modal description can be obtained from Eq.(1) by replacing the pressure with $p(x,y)e^{-ik_z z}$, as

$$\nabla^2\left(-\frac{1}{\rho_F}\right)p-\left(\frac{\omega^2}{v_F^2}-k_z^2\right)\frac{p}{\rho_F}=g, \qquad (2)$$

where k_z represents the longitudinal wave number in z direction.

By solving Eq.(2) at a given frequency with a nonzero excitation, one can obtain the most axial wave numbers. These values are the propagation constants of evanescent (stoneley) or propagating (rayleigh) waveguide modes. The frequency related to each mode is the modal or cut-off frequency of the guided path. It is the critical frequency between propagation and attenuation, which can be obtained as [5]

$$f_c=\frac{1}{2\pi}\left[\omega^2-k_z^2 v_F^2\right]^{1/2}. \qquad (3)$$

Theoretically, the modal frequency of fluid-filled cylindrical pipe with infinite stiffness surface can be calculated as [5]

$$f_c=\frac{v_F z_{mn}}{2\pi r} \text{ with } J'_m(z_{mn})=0, \qquad (4)$$

where r is the radius of the pipe, z_{mn} is the n-th root of the first derivative of Bessel function J'_m.

3. NUMERICAL IMPLEMENTATION

Let us consider a uniform and smooth fluid-filled PCCP surrounded by the outer formation as shown in Fig.1. For simplicity, assume the pipe is filled with static fluid (water) and damping is absent in all medium.

The model is simulated for the rigid and the elastic pipe with different radii. In this paper, the simulated results of rigid pipe are presented and the results are compared with the analytical solutions using Eq.(4). Similar analysis is also performed for elastic pipes, where analytical solutions are unknown. However, the simulated results are analyzed and validate based on the theoretical concept.

The following values of the parameters have been used for the numerical study: density of water, 997 kg m^{-3}; acoustic wave speed, 1500 m s^{-1}; inner radius varies as, R = 0.5m, 0.7m and 1.0 m, respectively. To avoid the computational burden, the solutions up to 3.5 kHz were evaluated.

4. RESULTS AND DISCUSSION

The numerical results of cut-off frequency obtained from simulated and analytical solutions are shown in Table-1, for various radii (R = 0.5m, 0.7m and 1.0 m) rigid pipe. From the results it is seen that, increasing the inner radius reduces the cut-off frequency of rayleigh modes, which also satisfies the analytical solution. In this work, the analysis is computed up to 6-th mode of propagation and found only 0.2 to 0.5% variations in the simulated results when compared to analytical solutions. And hence, the numerical solutions provide acceptable results to a reasonable level of accuracy.

The results easily identify the various propagating modes, their excitation frequency as well as the impact of the pipe radius. It is seen that, the number of propagating modes decreases with decreasing pipe radius, and vice versa. This signifies that, in the lower diameter pipe it is difficult to detect the low frequency acoustic waves by placing the sensors at far distance.

5. CONCLUSIONS

The impacts of the rigid pipe radius on the stoneley and the rayleigh modes of the system are described. The modes of propagation and cut-off frequency of acoustic wave in rigid pipes with a range of radii are observed.

The modal analysis is also useful to determine the modes of propagation for elastic pipe with complicated structure and medium, where direct analytical solution might not be known. This information is important for the placement of sensors in AE monitoring system to locate the corroded areas.

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Table 1. Analytical and simulated results of cut-off frequency of the rigid pipe.

Mode of Propagation (m, n)	Analytical, f_c (Hz)	Simulated, f_c (Hz)	Analytical, f_c (Hz)	Simulated, f_c (Hz)	Analytical, f_c (Hz)	Simulated, f_c (Hz)
	R = 0.5 m		R = 0.7 m		R = 1.0 m	
0,0	0.00	0.00	0.00	0.00	0.00	0.00
1,1	879.01	879.10	627.87	627.92	439.51	439.55
2,1	1458.18	1458.34	1041.56	1041.64	729.09	729.14
0,1	1829.65	1829.64	1306.55	1306.81	914.58	914.75
3,1	2005.83	2006.15	1432.74	1432.84	1002.91	1002.96
4,1	2538.68	2539.71	1813.34	1813.68	1269.34	1269.49
1,2	2545.37	2546.34	1818.12	1818.41	1272.68	1272.81
5,1	3062.94	3065.05	2187.81	2188.40	1531.47	1531.68
2,2	3201.88	3204.09	2287.06	2287.56	1600.94	1601.05
0,2	3349.89	3352.58	2392.44	2393.25	1674.71	1674.94
6,1	3581.46	3585.40	2558.19	2559.10	1790.73	1790.94