AN ENHANCED NUMERICAL MODEL TO SIMULATE NONLINEAR CONTINUOUS WAVE ULTRASOUND FIELD

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1. INTRODUCTION

Within the realm of therapeutic ultrasound, high intensity focused ultrasound (HIFU) is a rapidly expanding modality with applications in tumor necrosis, hemostasis and immunotherapy [1]. In this method of treatment highly focused ultrasound beams induce a rapid temperature rise around the focal spot due to conversion of acoustic energy to heat. Precise, well defined lesions can be created inside the tissue due to thermal coagulation. One advantage of HIFU over other similar treatment modalities is that it can be performed noninvasively. Selecting the right transducer and excitation parameters ensure that underlying tissue layers remain intact and tissue coagulation happen only around the focal spot.

Due to high acoustic pressure amplitude and intensity produced in focal region, a significant nonlinear distortion can be observed and thus an accurate propagation model needs to include the effect of nonlinearity [2] The model that we present here is based on a second-order operator splitting method where the acoustic field is propagated over incremental steps taking into account the effects of diffraction, nonlinearity and attenuation. This model is in essence a modified version of the KZK model where the parabolic diffraction term is replaced by a more accurate full diffraction term. This method was first introduced by Christopher et al. [3] for axi-symmetric sources and then improved by Tavakkoli et al. [4] via implementing larger propagation steps. It was then extended by Zemp et al. [5] to general non axi-symmetric problems using angular spectrum method.

In this work, we'll be further refining this method by introducing arbitrary source geometry and excitation definition, full diffraction solution, enhanced pressure calculation, and enhanced power deposition rate and temperature prediction capabilities. The result is a particularly useful tool in carrying out simulations of HIFU beams in tissue including temperature rise predictions. Since a typical HIFU power is usually delivered for the duration of a few seconds at frequencies of a few MHz, a CW simulation will be suitable.

2. METHOD

The KZK equation, which accounts for combined effects of diffraction, attenuation and nonlinearity in propagation of acoustic beam, is given in Eq. (1) below:

$$\frac{\partial p}{\partial z} = \frac{c_o}{2}\int_{-\infty}^{\tau} \nabla_{\perp}^{2} p \, d\tau + \frac{1}{2c_o^3 \rho_o}\left[\left(\mu_B + \frac{4}{3}\mu\right)\frac{\partial^2 p}{\partial \tau^2} + \beta \frac{\partial p^2}{\partial \tau}\right] \quad (1)$$

The first term on the right hand side is the diffraction term in parabolic approximation, the second term reflects the effect of attenuation and the third term is due to nonlinearity. The pressure field can be calculated over propagations planes in incremental steps by bringing $\frac{\partial p}{\partial z}$ to the left side as in Eq. (1). Also based on the above equation, the effects of diffraction, attenuation and nonlinearity can be applied independently over propagation planes and then added together. This is often referred to as operator splitting method. In the second-order operator splitting method, a certain propagation scheme is maintained which enable larger propagation steps and faster computational time (see Fig.1) [4].

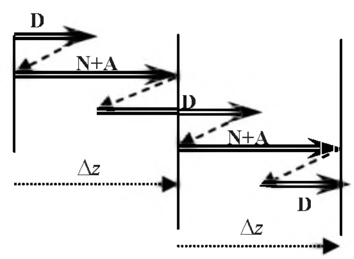

Figure 1. Second-order operator splitting method. D, N, and A represent operators for diffraction, nonlinearity and attenuation respectively.

Note that in this method, nonlinearity and attenuation are combined and propagated in one step. For a CW periodic waveform, the equations of propagations in each step shown in Figure 1 are presented here. For diffraction over the nth harmonic:

$$v_z(x, y, z + \Delta z) = \Im_{2D}^{-1}\left\{\Im_{2D}\left\{v_z(x, y, z)\right\} \times H(k_x, k_y, \Delta z)\right\} \quad (2)$$

where $H(k_x, k_y, \Delta z) = e^{j\Delta z\sqrt{k^2-(k_x^2+k_y^2)}}$ and $k = 2\pi(nf_o)c_o$ and k_x, k_y are spatial frequency components. This will be repeated over N harmonics ($n = 1$ to N).

For nonlinearity and attenuation over the nth harmonic:

$$v_n(z+\Delta z) = v_n(z) + j\frac{2\pi\beta f_o}{2c_o^2}\Delta z\left[\sum_{i=1}^{n-1} i v_i v_{n-i} + \sum_{i=n+1}^{N} n v_i v_{n-i}^{*}\right] - \alpha_o (nf_o)^b v_n \Delta z \quad (3)$$

which will be repeated over N harmonics as well.

In the enhanced version of the algorithm, user has the capability to define an arbitrary source geometry and input excitation. Since the propagation is done plane by plane, this will require an extra initial step to propagate the field from the surface of transducer to an initial plane. This step is done using the Rayleigh diffraction integral which assumes linear propagation from the source to the first plane. Another method to accomplish this is to propagate the beam from the source onto the initial plane by introducing simple phase shifts. Phase shift methods, however, produce inaccurate results in near field specially when the source surface is highly focused

The normal particle velocity is then calculated on equally spaced discrete points across the initial plane (intersection of solid lines in Fig. 2). The calculated values of v_z is then expanded and assigned to the adjacent squares (dotted lines) to create a 2D array as shown in Fig. 2.

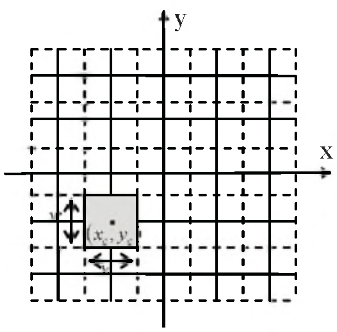

Figure 2. Initial plane as a 2D array.

Since the value of v_z across any given square (e.g. the shaded area shown in Fig. 2) is constant, it can be written in compact form as $v_z = v_o \mathrm{rect}\left(\frac{(x-x_c)}{w}, \frac{(y-y_c)}{w}\right)$ where w is the width of the array element and (x_c, y_c) is the location of the element's centre. The *rect* function has an analytical 2D Fourier transform as below:

$$\Im_{2D}(v_z) = v_o w^2 \mathrm{sinc}(w\frac{k_x}{2\pi}, w\frac{k_y}{2\pi}) \times e^{-j(k_x x_c + k_y y_c)} \qquad (4)$$

Eq. 4 is then added up across all array elements to calculate the Fourier transform of the entire plane. The result is then feed into Eq. 2 to perform the first half step diffraction as illustrated in Fig. 1. After finishing diffraction substep**,** the result is then converted back to spatial domain using inverse Fourier transform and a nonlinear substep is subsequently performed using Eq. 3. The process is then repeated to propagate the field along the *z* direction.

3. RESULTS

The results obtained using our method were compared with other methods both in linear and nonlinear regimes. In overall, excellent agreements were observed. Fig. 3 displays lateral pressure profiles for a concave spherical transducer with effective radius of curvature of 160 mm and aperture diameter of 37.6 mm working at a frequency of 2.25 MHz and with a source pressure of 92.5 KPa. Our results are in excellent agreement with those obtained by Averkiou et al. [6] using the KZK nonlinear model as shown in Fig. 3.

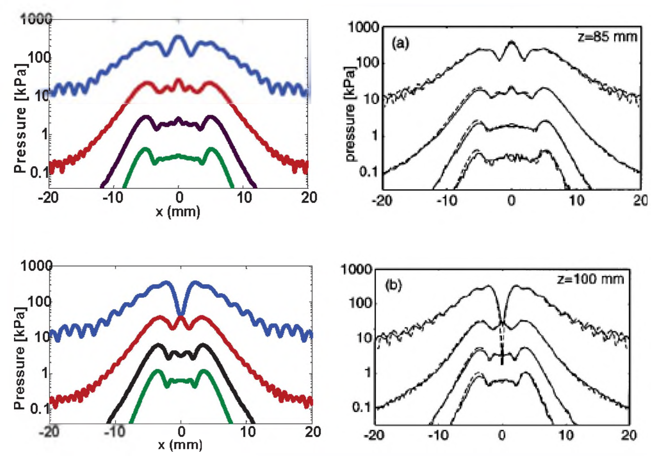

Figure 3. Our simulation results (left column) vs. the KZK nonlinear model (solid and dashed lines represent measurement and simulation results, respectively), for the fundamental and first 3 harmonics.

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ACKNOWLEDGEMENTS

This work was partially supported by the Dean's Start-up Fund from Ryerson University.