EXPERIMENTAL STUDIES ON THE IN-PLANE VIBRATIONS AND SOUND RADIATION IN AN ANNULAR THICK DISK

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1. INTRODUCTION

Studies on the in-plane vibrations and sound radiation in circular annular disks are rarely found in literature. Most of the studies are on the out-of-plane vibrations since the contribution from the in-plane vibrations are small compared to out of plane vibrations. However, in thick plates, modes of vibration within the plane of the disks are coupled with the out-of-plane modes, contributing to total vibration and noise radiation (Tzou et. al (1998)). Due to this coupling, out-of-plane modes can be excited by forces along the plane of the disk. Moreover, sound radiation from in-plane modes is significant in many engineering applications such as disk brakes and railway wheels (Thompson (2000)).

Non-uniformity in the boundary conditions affects both the in-plane and out-of-plane natural frequencies of the disk. Studies on the out-of-plane vibrations show that some of the natural frequencies have different values for the symmetric and anti-symmetric modes. This observation has been discussed based on analytical and experimental investigations (see for example Bauer and Eidel (2006) and Eastep and Hemmig (1982)). Experimental studies on the in-plane vibrations in circular annular disks subject to non-uniform boundary conditions are not found in literature. Moreover, previous studies on the in-plane vibrations were obtained for disks with uniform boundary conditions only. Review of previous analytical studies on the subject can be found in Bashmal et al. (2008). Few experimental investigations were conducted on in-plane vibrations of circular disks with uniform boundary conditions. Ambati et al. (1976) measured the in-plane vibrations of free annular disks with different radius ratios to study the characteristics of vibrating modes as the annular disk approaches thin ring. The natural frequencies and sound pressure from in-plane vibrations were measured for annular disks with free boundary conditions by Lee (2003). Leung and Pinnington (1987) conducted experiments to measure the in-plane response of stationary disk under rotating edge loads.

The present study presents experimental investigations on the in-plane vibrations of thick plates with non-uniform boundary conditions. The natural frequencies of an annular disk supported at one point are measured and compared with values measured for free support conditions. Moreover, using the Rayleigh-Ritz method, the boundary characteristic orthogonal polynomials are used as admissible functions in a three dimensional analysis to obtain the natural frequencies and the associated mode shapes analytically.

2. THEORY

In this section, a three-dimensional thick disk model is used to investigate the modal characteristics of a stationary circular disk using the Rayleigh-Ritz method. The material of the disk is assumed to be isotropic with mass density ρ, Young's modulus E and Poisson ratio ν. Let the outer radius of the disk be R_o, the inner radius be R_i and the thickness of the disk be h. The radial, circumferential and transverse displacement components of a point in the annular disk are denoted by u_r , u_θ and u_z , respectively. Introducing the non-dimensional parameter, $\xi=r/R_o$, $\zeta=z/h$ the expression for the maximum kinetic (T_{max}) and strain (Π_{max}) energies of the disk in cylindrical coordinates (ξ,θ,ζ) are(So and Leissa (1998)):

$$\begin{aligned}\Pi_{max} = \frac{1}{4}\frac{Eh}{(1+\nu)}\int_0^{2\pi}\int_{-0.5}^{0.5}\int_{\beta}^{1}\Bigg\{&\frac{2\nu}{(1-2\nu)}\left(\frac{\partial u_r}{\partial\xi}+\frac{1}{\xi}\frac{\partial u_\theta}{\partial\theta}+\frac{u_r}{\xi}\right.\\ &\left.+\frac{R_0}{h}\frac{\partial u_z}{\partial\zeta}\right)^2+2\left[\left(\frac{\partial u_r}{\partial\xi}\right)^2+\left(\frac{u_r}{\xi}+\frac{1}{\xi}\frac{\partial u_\theta}{\partial\theta}\right)^2\right.\\ &+\left.\left(\frac{R_0}{h}\frac{\partial u_z}{\partial\zeta}\right)^2\right]+\left(\frac{1}{\xi}\frac{\partial u_r}{\partial\theta}+\frac{\partial u_\theta}{\partial\xi}-\frac{u_\theta}{\xi}\right)^2\\ &+\left(\frac{R_0}{h}\frac{\partial u_\theta}{\partial\zeta}+\frac{1}{\xi}\frac{\partial u_z}{\partial\theta}\right)^2+\left(\frac{R_0}{h}\frac{\partial u_r}{\partial\zeta}+\frac{\partial u_z}{\partial\xi}\right)^2\Bigg\}\xi d\xi d\zeta d\theta\end{aligned} \quad (1)$$

$$T_{max} = \frac{1}{2}\omega^2 h\rho R_0^2\int_0^{2\pi}\int_{-0.5}^{0.5}\int_{\beta}^{1}(u_r^2+u_\theta^2+u_z^2)\xi d\xi d\zeta d\theta \quad (2)$$

The free in-plane vibration of the disk is assumed to have sinusoidal variations along the circumferential direction of the disk, and may be expressed in the form:

$$\begin{aligned}u_r(\xi,\zeta,\theta) = \sum_{l=0}^{\infty}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\big[&\bar{U}_{c,lmn}(t)\psi_l(\zeta)\phi_m(\xi)\cos(n\theta)\\ &+\bar{U}_{s,mn}(t)\psi_l(\zeta)\phi_m(\xi)\sin(n\theta)\big]\end{aligned} \quad (3)$$

$$\begin{aligned}u_\theta(\xi,\zeta,\theta) = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty}\sum_{k=0}^{\infty}\big[&\bar{V}_{c,ijk}(t)\psi_i(\zeta)\phi_j(\xi)\cos(k\theta)\\ &+\bar{V}_{s,ijk}(t)\psi_i(\zeta)\phi_j(\xi)\sin(k\theta)\big]\end{aligned} \quad (4)$$

$$\begin{aligned}u_z(\xi,\zeta,\theta) = \sum_{p=0}^{\infty}\sum_{q=1}^{\infty}\sum_{g=0}^{\infty}\big[&\bar{W}_{c,pqg}(t)\psi_p(\zeta)\phi_q(\xi)\cos(g\theta)\\ &+\bar{W}_{s,pqg}(t)\psi_p(\zeta)\phi_q(\xi)\sin(g\theta)\big]\end{aligned} \quad (5)$$

where $\overline{U}$, $\overline{V}$ and $\overline{W}$ are radial, circumferential and transverse deflection coefficients, respectively. The subscripts, c and s refer to cosine and sine components of the deflections, respectively. The function $\phi(\xi)$ and $\psi(\zeta)$ are the assumed deflection shape satisfying the geometric boundary conditions in the form of boundary characteristic orthogonal polynomials (Bhat (1985)). The assumed solutions - equations (3) to (5)- are substituted into the energy equations (1) and (2) to predict the natural frequencies using the Rayleigh-Ritz method.

3. EXPERIMENT

An experiment for measuring the natural frequencies of an annular disk subject to different boundary conditions is performed. The disk used in the experiment is an annular steel disk of 300 mm outer diameter, 100 mm inner diameter and 10 mm thickness. An accelerometer is mounted on the outer edge of the disk to capture the acceleration in the radial direction.. An accelerometer is mounted on the face of the disk to measure the out-of-plane accelerations. The disk is excited by an impulse hammer along the radial direction. The natural frequencies are measured for the disk supported horizontally on soft cushion to resemble free boundary conditions. Then, the disk is mounted on a vise along portion of its outer edge to represent a point support. The frequency range considered in the experiment is 12 kHz to capture the first four in-plane natural frequencies.

Table 1: In-plane natural frequencies of an annular disk with free edges ($\beta = 0.3, v = 0.3$).

Experimental (Hz)	4848	8952	9720	10450
Analytical (Hz)	4821	8865	9602	10332

4. DISCUSSION

The natural frequencies obtained analytically are compared with those measured in the experiment. Table 1 shows the in-plane modes of vibration of a free disk while Table 2 shows the out-of-plane modes of vibration. Out-of plane modes were measured to distinguish the in-plane modes from out-of-plane modes detected due to coupling between modes. Comparison shows good agreement between analytical and experimental results. The frequency spectrum of the in-plane modes of vibration is also show in Figure 1. For the disk mounted on a vise, the frequency spectrum, shown in Figure 2, shows that some of the modes split into two different values. This can be attributed to the nature of coupling between different cosine and sine modes due to the non-uniformity of the boundary conditions.

Table 2: Out-of-plane natural frequencies of an annular disk with free edges ($\beta = 0.3, v = 0.3$).

Experimental (Hz)	522	936	1326	1908	2354	3486	3598
Analytical (Hz)	528	922	1337	1965	2388	3572	3678

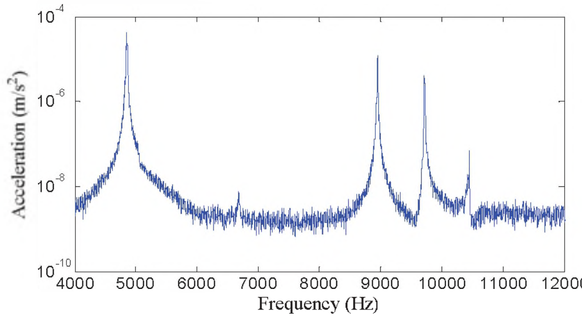

Figure 1: Frequency spectrum for an annular disk with free edges.

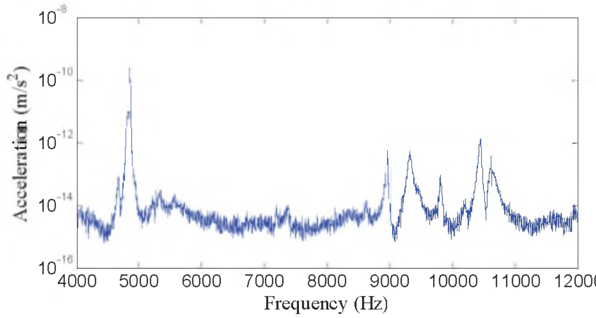

Figure 2: Frequency spectrum for an annular disk with point support at outer edge.

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