MODELING ELECTROSTATIC FIELD IN MEMS DEVICES USING ARTIFICIAL SPRINGS IN RAYLEIGH-RITZ METHOD

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1. INTRODUCTION

Micro Electro Mechanical Systems (MEMS) are mechanical devices having moving components of size in the order of tens of micrometers that are integrated with electronic circuitry in the same chip. An electrostatically actuated MEMS device is shown Fig. 1. This device was manufactured through surface micromachining process.

Structural modification of an elastic system such as introducing stiffeners or creating notches would affect the dynamic behavior of the system. The operational environment such as the electrostatic field acts in the form of an elastic foundation and influences the stiffness property of the structure in addition to the structural geometry, support conditions, fabrication process [1-4]. The boundary support conditions obtained for microfabricated systems are not classical and can be somewhere in between pinned and clamped conditions. A method to quantify the above effects will be very useful in designing and modeling the dynamic performance of the system.

Bhat [5] presented a method to obtain the natural frequencies of rectangular plates under various boundary conditions using boundary characteristic orthogonal polynomials in the Rayleigh Ritz method. These orthogonal polynomials were generated using Gram-Schmidt process. The first polynomial is chosen so that it satisfies at least the geometrical boundary conditions of the structure.

In the present work, the plate type microdevice, boundary conditioned with external electrostatic field, has been analyzed for its dynamic behavior using boundary characteristic orthogonal polynomials in the Rayleigh-Ritz method for different end support conditions, namely, clamped (CCCC), simply supported (SSSS) and microfabricated condition (MMMM) that lies between the CCCC and SSSS conditions. A formulation to obtain the static deflection of the microdevice is also presented in this paper. Further, the vibration problem is formulated at the static equilibrium condition by including an elastic foundation stiffness to represent the electrostatic field effects in order to obtain the modeshapes and the Eigenvalues of the plate type microdevice at the static equilibrium position.

2. MODELING USING ARTIFICIAL SPRINGS

The microdevice is supported by translational and rotational springs with stiffness values ‘K_T’ and ‘K_R’ per unit length at the end supports. The microdevice is the main electrode which is separated by a gap of ‘d’ from the substrate that acts as the bottom electrode. When a bias voltage ‘V’ is applied between the device and the substrate, electrostatic field is created [6] and it results in attractive electrostatic force that pulls the plate type main electrode towards the substrate.

This electrostatic field has a softening effect on the elastic property of the system, since it acts in opposition to the structural stiffness. In order to apply the concept of boundary conditioning to represent the electrostatic effect, quantification of the softening effect in terms of distributed springs or an elastic foundation with stiffness per unit area of ‘Ke’ is introduced [7]. As the deflection increases due to increase in voltage, the value of the stiffness ‘Ke’ will indicate more weakening.

It is known that a constant value of the stiffness Ke(x, y) would result in different natural frequencies but same modeshapes when compared to the microdevice without elastic foundation. As the deflection of the microdevice due to electrostatic force results in non-uniform spacing between the electrodes, it results in softening elastic foundation with non-uniform variation of Ke(x, y) as shown in Fig.2. Even though, the electrostatic field results in non-linear dynamic behavior, it is linearly represented with artificial springs as most of the electrostatically operated MEMS devices are in the linear operating range. The static behavior shown in Fig. 3 has been predicted with energy method while the dynamic behavior has been predicted with Rayleigh-Ritz method.

3. RESULTS AND DISCUSSION

The dimensions of the microdevice are 600μm in length and breadth for a constant electrode gap of 15μm. Fig. 4 shows the variation between ‘V’ and eigenvalue of the first mode for increasing voltage. Eigenvalue for each support condition reaches zero for a certain voltage called snap voltage as shown in Fig. 4. The graph indicates that the snap voltage of the square microplate for MMMM condition lies between the CCCC and SSSS condition. The pull-in roughly occurs when the microdevice reaches 1/3 of the gap between the two electrodes. Using snap voltage as a basis, the microdevice has been analysed using a normalized voltage parameter defined as,

$$\sigma = \left(\frac{V}{V_{snap}} \right) \times 100\% \qquad (1)$$

where, 'V_{snap}' is the pull-in voltage for the corresponding end condition and 'V' is the applied voltage. This normalized parameter is used to study the effect of electrostatic field on the modeshapes and natural frequencies of the microdevice. As the voltage is increased the eigenvalue decreases.

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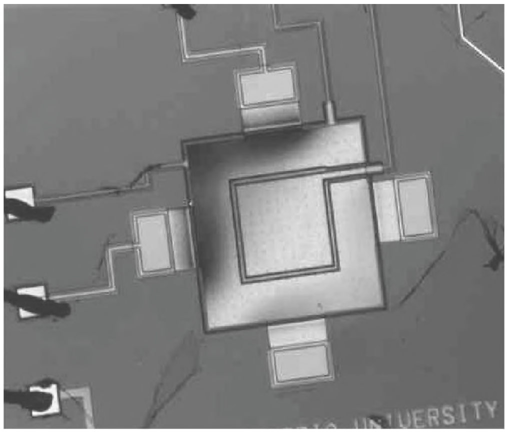

Figure 1 Surface Micromachined Capacitive Microdevice

Size of the microplate: 600μm x600μm

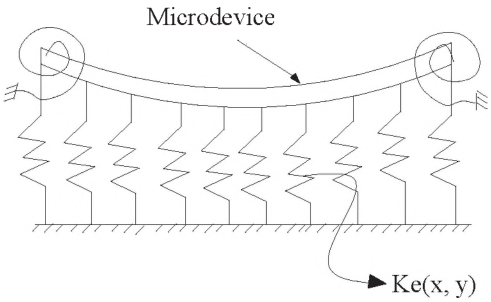

Figure 2 Electrostatic field modeled as non-uniformly distributed spring

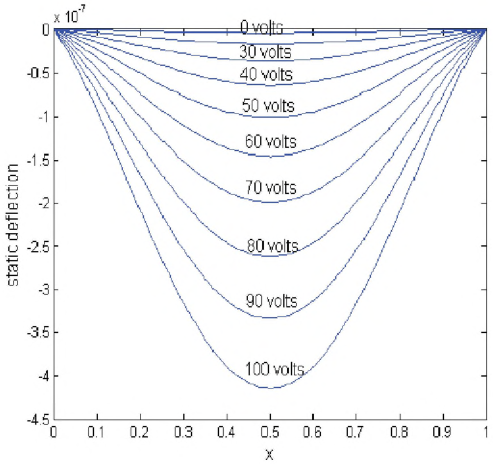

Figure 3 Static deflection in μm at y=0.5 for MMMM condition at different voltages.

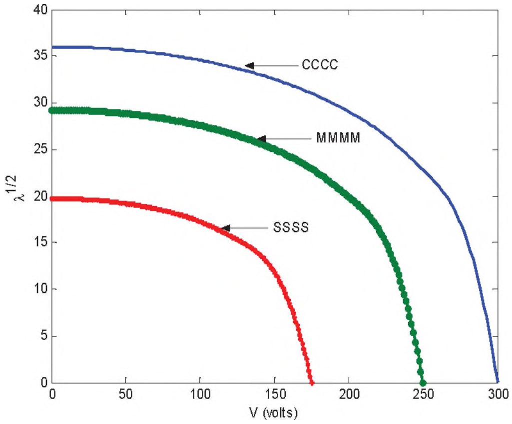

Figure 4 Variation of fundamental eigenvalues against bias voltag indicated end support conditions