DETERMINING ACOUSTIC SCATTERING PROPERTIES OF MARINE SEDIMENTS THROUGH BAYESIAN INVERSION

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1. INTRODUCTION

Reverberation modeling and sonar performance prediction in shallow waters require good estimates of seabed reflection and scattering properties as well as an understanding of scattering processes in a particular region. This paper describes non-linear Bayesian inversion of synthetic ocean acoustic seabed scattering data for marine sediment parameters, assuming first-order perturbation theory for acoustic scattering. The objective is to determine the necessary angular range to adequately determine the model parameters

2. METHOD

There are three elements in an inversion; these are the data, the forward model (physics), and the inversion scheme. This section gives a brief description of all three of these elements.

2.1 Inversion scheme

Bayesian inversion is based on formulating the posterior probability density (PPD) of the model parameters of interest, which is the product of likelihood and prior information or distribution of the parameters [1,2]. The PPD contains all available information for the parameters; however it can be difficult to interpret directly in an analytic manner. Thus the PPD is approximated using Markov-chain Monte Carlo sampling algorithms. This approximate PPD is then interpreted in terms of its moments, parameter uncertainties (variances, marginal distributions, credibility intervals), and parameter inter-relationships (correlations and joint marginal's). The Bayesian formulation also provides information measures which quantify the evidence provided by observed data to support a particular choice of model parameterization and/or forward (modeling) theory, favoring the simplest choice consistent with the resolving power of the data (Bayesian form of Occam's razor).

2.2 Forward model

In the present application the forward model used describes the backscatter emitted from an insonified rough boundary between two otherwise homogenous half spaces. The configuration is shown in Fig. 1.

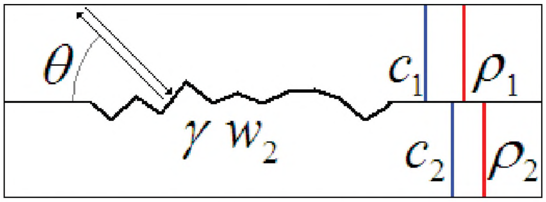

Fig. 1: A schematic of the forward model for acoustic backscatter from a rough interface.

First-order perturbation theory is used to generate and invert the synthetic data [3]. The scattering kernel used (ξ) is given in Equation 1, where k_1 is the wave number of the first medium, and θ is the grazing angle for both thin incident and scatter rays. The scattering exponent is γ, and the spectral strength is w_2.

$$\xi = \frac{4k_1^4 \sin^4(\theta)\left|R(\theta)\right|^2 w_2}{K(\theta)^\gamma} \tag{1}$$

The functions R and K are given in Equations 2 and 3, where ρ is the ratio of densities of the second to the first media, and κ is the ratio of wave numbers.

$$R(\theta) = \frac{(\rho-1)^2\cos^2(\theta) + \rho^2 - \kappa^2}{\left(\rho\sin(\theta) + \sqrt{\kappa^2 - \cos(\theta)^2}\right)^2} \tag{2}$$

$$K(\theta) = \sqrt{4k_1^2\cos^2(\theta) + \left(\frac{k_1}{10}\right)^2} \tag{3}$$

2.2 Synthetic data

Nine synthetic data sets were created using Equation 4. This is the decibel representation of Equation 1. A calibration bias term β is also decibel added. This will allow for the inversion of bias data when working with non synthetic data.

$$d_i = 10\log_{10}(\xi_i) + \beta + \varepsilon_i \tag{4}$$

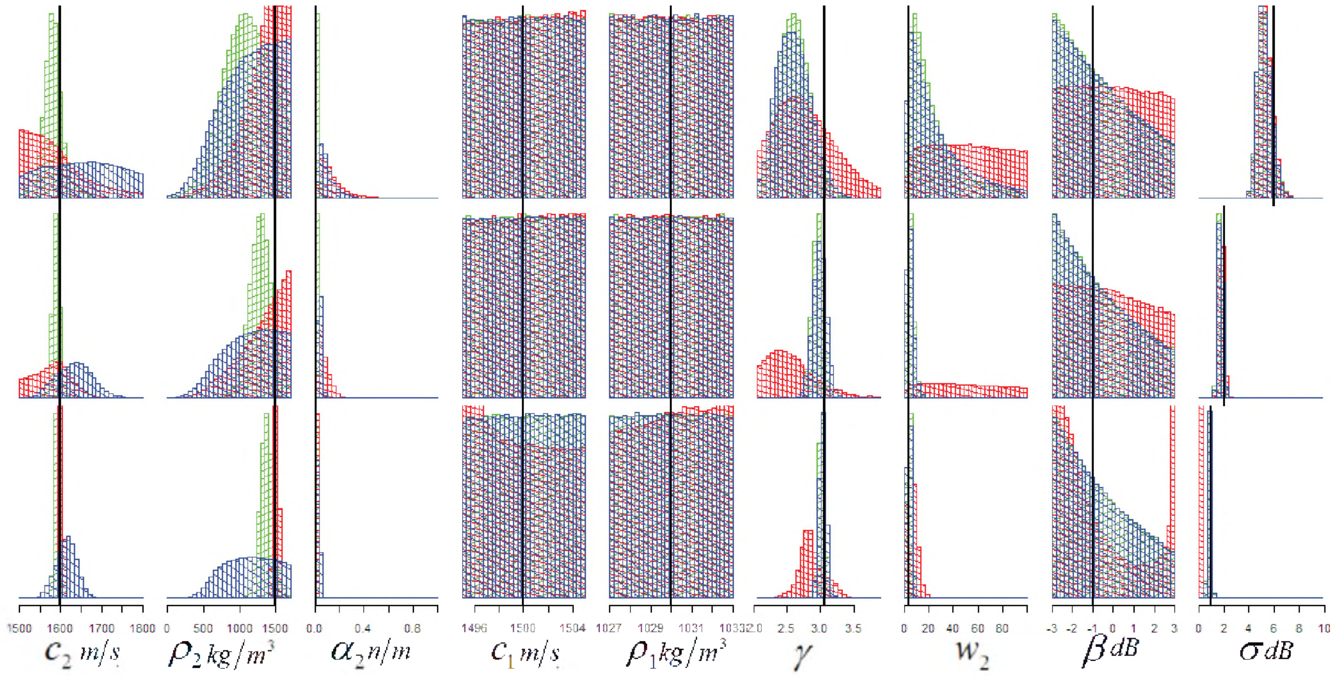

Fig. 2: Marginal distributions for all nine parameters sampled from the PPDs of the data sets. The three rows from top to bottom correspond to data error standard deviations (σ) of 6, 2 and 1dB respectively. The true value is displayed as a black line. 🟥 for θϵ[1,21], 🟦 for θϵ [22,89], and 🟩 for θϵ [1,89].

Each data set contains 45 data points (d_1, d_2, …, d_{45}). These data sets differ in that they consider different ranges of θ and different standard deviations for the Gaussian error terms ε . The ranges of the data are 1-89°, 1-21°, and 22-89°. The standard errors (σ) of the ε s are 6, 2, or 1 dB depending on the data set.

The true values of the nine parameters are $c_2 = 1600$, $\rho_2 = 1500$, $\alpha_2 = 0.02$, $c_1 = 1500$, $\rho_1 = 1030$, $\gamma = 3.06$, $w_2 = 3.6$, $\beta = -1$, and $\sigma = 1,2 \vee 3$.

3. RESULTS

The one-dimensional marginal PPDs for the parameters are shown in Fig. 2. Each row contains nine histograms; one for each of the parameters of interest. The rows are sorted in descending order according to the different values of σ, 6, 2 and 1 dB. Each histogram plot displays three marginal PPDs for the given parameter. They are distinct in that they are estimated from different data sets. The data sets were created using the same true values for the parameters, but with different angular ranges. Thus a total of 81 marginal distributions are presented. The true value of the parameters is displayed as the vertical black line across each histogram.

4. DISCUSSION

As expected, the PPDs have smaller variances (are narrower) as the data errors are reduced. The data sets that consider only θ below the critical angle (21°) are not adequate to estimate the scattering parameters (γ, w_2) because the marginal distributions are too wide; that is, they contain little information. As surface scattering will not dominate above this angle, to estimate these parameters requires more information be added to the inversion. For example, reflection data could be added and a joint inversion performed. An alternative approach would be to consider a more complex scattering kernel that accounts for sub-bottom or volume scattering.

REFERENCES

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