

# MULTIPLE SHOCK LOADING ON FLUID-FILLED SHELL STRUCTURES

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## 1. INTRODUCTION

The analysis of the scenario where a submerged shell structure is subjected to two or more consecutive shock waves is of definite practical interest, one of the most obvious examples being a complex shock loading generated as a result of the reflection of the original shock wave off one or more reflective surfaces in the proximity of the structure. At the same time, the relevant published studies of such multiple loading scenarios are quite scarce, perhaps owing to the fact that there are a number of higher-priority shock-structure interaction problems that over the years attracted more attention from the research community and more funding from the industry.

More specifically, although the somewhat relevant complex reflection patterns in systems comprised of several components were considered in a number of studies (e.g. [1,2]), it appears that, to the best of our knowledge, the interaction between a submerged fluid-filled shell and a multi-front shock wave has not yet been considered.

Our goal, therefore, is to report some preliminary findings for a fluid-filled submerged cylindrical shell subjected to an external shock wave with two fronts (or, equivalently, to two consecutive shock waves). We are particularly interested in developing an understanding of the structure and evolution of the internal hydrodynamic field induced by such loading since the internal hydrodynamic loading will determine some of the peak values of both the pressure and structural stress.

## 2. MATHEMATICAL FORMULATION AND SOLUTION METHODOLOGY

We consider a thin elastic circular cylindrical shell filled with and submerged into identical fluids. We assume that the shell is thin enough, and that its deflections are small in comparison to its thickness, so that the linear shell theory can be employed; we further assume that the Love-Kirchhoff hypothesis holds true.

The fluids are assumed to be irrotational, inviscid, and linearly compressible, thus the wave equations are used to model the fluid dynamics. The fluids and the shell are coupled through the dynamic boundary condition on the interface.

The problem is approached with the methodology we developed in our earlier work [3, 4], i.e. we apply the Laplace transform time-wise to the wave equations and then separate the spatial variables in order to arrive at the expressions for the transforms of the internal and external pressure in a form of a series of modified Bessel functions of the first (internal fluid) and second (external fluid) kind. Upon inverting the transforms we obtain the pressure as a Fourier series with time-dependant coefficients which, for the radiation pressure, still depend on the unknown at this stage normal displacements of the shell.

Then, we use the same series form for the shell displacements and, substituting them into the shell equations, we arrive at the systems of the ordinary differential equations for each of the displacement harmonics. The systems are then approached numerically (finite differences) and the resulting normal displacement is used in the pressure series to compute the radiation pressure everywhere around the shell.

The newest addition to the approach is incorporating a shock wave with multiple fronts, more specifically, a shock loading comprised of two consecutive plane shock waves of the same intensity; the equations and relevant discussion for the individual waves can be found in [4].

We note that although this model is quite simple in that any realistic shock wave being reflected off any surface will generate another shock wave with a different location of its 'virtual source', with a plane wave being the only exception in some instances, we still think that the results we present here are of value since they establish the basic framework for analysis of multiple shock loading, and since they allow us to highlight the most important phenomena that make the present case so different from its single-wave counterpart.

The methodology we use has been extensively (and successfully) verified for weak shock waves and acoustic pulses in our earlier work [3,4], and since the present loading does not differ from the single-front shock waves used in those studies in terms of its intensity, we are convinced that the methodology can be successfully used to address the problem in question as well.

## 3. RESULTS AND CONCLUSIONS

A steel shell is considered with the thickness of 0.01 m and radius of 1 m, submerged into and filled with water. The interaction with two identical consecutive plane incident waves is analyzed, with the rate of exponential decay of 0.0001314 s and the initial pressure in the front of 250 kPa. Although the model allows for the consideration of any distance between the wavefronts to be considered (which, of course, corresponds to different delays of the offset of the second wave), we only consider a single value of the dimensionless delay of 0.9 (the dimensionless unity is equal to the time it takes for the shock wave to pass over the radius of the shell). This value means that the second wave arrives at the shell when the first one is just about to reach the axis.

Fig. 1 shows three numerically simulated sequential images of the internal field for the chosen shock wave during the early and middle interaction (the external field is not shown since we have observed that all the most interesting phenomena due to the presence of the second front are taking place in the internal fluid).

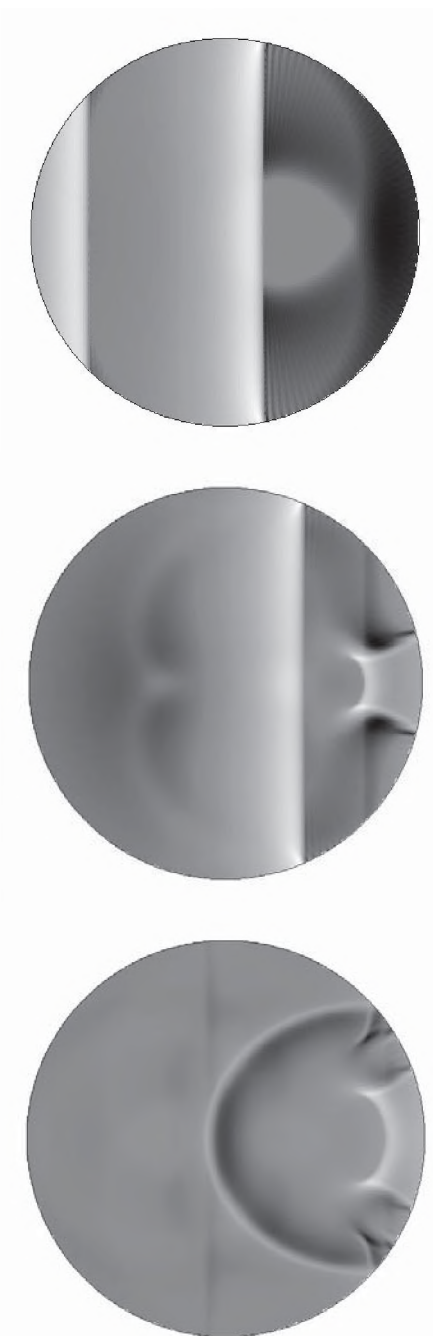


Figure 1. The internal hydrodynamic pattern at the dimensionless time instants of 1.20 (top), 2.30 (middle) and 3.10 (bottom).

The first image shows an instant when both waves has already impacted the shell and originated the respective internal pressure waves propagating downstream (from here on we will refer to the internal pressure waves simply as 'shock waves', minding the remarks on the matter found in [3]). In the second image, the first internal wave has reflected off the tail region and is forming the classical post-reflection pattern, but the second one is still propagating downstream, with its regular reflection pattern developing. In the third image, both waves have reflected and are propagating upstream, resulting in the field allowing one to simultaneously see two different stages of the development of the reflection pattern in one image (an effect somewhat

similar to the one seen in one of the experimental images of [5]; in that work, however, the presence of such a double structure was due to the overlaying of images taken at different instances, not due to the existence of an actual double-front wave).

As was expected, the structure of the field for the double-wave case is much more complex than that for the single wave one. Even the three representative illustrations shown here make a strong enough case for a thorough further investigation of the two-wave scenario. Specifically, we expect that the superposition of various features of the internal field will, in some cases, result in new features that are of considerable practical interest.

Investigating such effects is one of the future objectives of our research program, and it will require a more in-depth study of the interaction of a rather wide range of the time intervals between the shock waves. Such an investigation will also facilitate the future studies of systems where multiple surfaces are present. In particular, it will result in a better understanding of what mutual locations of the shell and the reflective surfaces should be given a priority in such studies.

## REFERENCES

- [1] Jialing, L. Hongli, N. (1997) Study on propagation of blast wave over cylinder in a channel bend, proceedings of the 21st International Symposium on Shock Waves, Great Keppel Island, Australia, July 1997, pp.103–107.
- [2] Oakley, J.G., Anderson, M. H., Wang, S., Bonazza, R. (2001) Shock loading of a cylinder bank with imaging and pressure measurements, proceedings of the 23rd International Symposium on Shock Waves, Fort Worth, Texas, USA, July 2001, pp.589–595.
- [3] Iakovlev, S. (2006) External shock loading on a submerged fluid-filled cylindrical shell, *Journal of Fluids and Structures* 22, 997-1028.
- [4] Iakovlev, S. (2008) Interaction between a submerged evacuated cylindrical shell and a shock wave. Part I: Diffraction-radiation analysis, *Journal of Fluids and Structures* 24, 1077-1097.
- [5] Sun, M. and Takayama, K. (1996) A holographic interferometric study of shock wave focusing in a circular reflector, *Shock Waves* 6, 323-336.

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