A NONLINEAR GEOMETRICAL ACOUSTIC MODEL FOR SONIC BOOM PROPAGATION

M. Berci[1] and L. Vigevano[2]
[1]iETSI, School of Mechanical Engineering, University of Leeds, UK, m.berci@leeds.ac.uk
[2]Dipartimento di Ingegneria Aerospaziale, Politecnico di Milano, Milano, Italy, luigi.vigevano@polimi.it

1. INTRODUCTION

Sonic boom prediction is a typical multi-scale problem, since the pressure signature generated by the shock structure at the length scale of the aircraft, L, in the "near-field", is transmitted through the atmosphere a long distance away, of the order of $100L$, to the "far-field". The propagation of the pressure perturbation is the most important aspect of the phenomenon: small amplitude nonlinear effects accumulate over long distances and distort the pressure signature significantly, giving rise to the coalescence of the pressure distribution into shocks and typically resulting in an asymptotic N-wave[1]. The goal of the present work is not to propose a new comprehensive theory of sonic boom propagation but rather to formulate an analytical model which can predict accurately, under limited conditions, the pressure signature on the ground in the vertical plane below an aircraft (where its sonic boom of maximum intensity lays, due to its minimum distance from the ground) in steady horizontal supersonic flight. To this aim, we propose to combine a revisited formulation of the nonlinear treatment of the pressure wave evolution due to Friedman and coauthors[2] with the simplified calculation of its nonlinear distortion due to George and Plotkin[3], supplemented by the "area rule" for the shock waves formation[4]; straightforward adaptations of the ray-tracing system obtained by Randall[5] and ray-tube area calculated by Pierce and Thomas[6] are then employed to complete the set of necessary equations, along with the standard atmosphere model. This combined method is simple and, although limited to a constant horizontal aircraft speed and a still atmosphere, allows a very accurate and efficient prediction of the boom propagation starting from a given pressure signature in the near-field.

2. METHOD

The sonic boom intensity at the ground $Z = 0$ is derived from an initial pressure signature generated by an aircraft in supersonic cruise flight at the altitude $Z = Z_f$ and then propagated along the acoustic ray-tubes through the real stratified atmosphere. Expanding the solution of 1D Euler equations as an isentropic perturbation of the atmospheric conditions[2], it is possible to separate the spatial dependence from the time dependence of the boom propagation problem and hence to investigate the different orders of its solution. The zeroth-order terms of the mass and momentum equations lead to:

$$P_1 = \frac{\gamma P_0}{a_0} w_1 = a_0 \rho_0 w_1, \tag{1}$$

while considering the first-order terms we obtain the following Bernoulli/Riccati ODE:

$$\frac{dw_1}{ds} + \frac{1}{2}\left(\frac{1}{A}\frac{dA}{ds} + \frac{1}{P_0}\frac{dP_0}{ds} - \frac{1}{a_0}\frac{da_0}{ds}\right) w_1 - \frac{(\gamma+1)}{2a_0^2} w_1^2 = 0. \tag{2}$$

If we neglect the second-order perturbation velocity term in the equation above, we recover the solution presented in [2] that we call here the quasi-linear model, which accounts for the variation of both the acoustic impedance $a_0\rho_0$ and the ray-tube area A through the atmosphere:

$$\Delta P_{ql} = \sqrt{\frac{a_0 \rho_0 A_h}{a_h \rho_h A}}\, \Delta P_h, \tag{3}$$

where subscript h refers to the distance r_h from the aircraft where the pressure signature is assigned. If we consider the whole ODE, we have a nonlinear theory (the best that can be done within the assumption of isentropic flow) which results in:

$$\Delta P_{nl} = \frac{\sqrt{\frac{a_0 \rho_0 A_h}{a_h \rho_h A}}}{1 + \frac{\gamma+1}{2\gamma}\sqrt{\frac{P_h a_h^2}{a_h s_h}}\left|\left(\frac{\Delta P}{P}\right)_h\right| \int_{s_h}^{s} \sqrt{\frac{a_0 A_h}{P_0 A}}\frac{ds}{a_0^2}}\, \Delta P_h. \tag{4}$$

In order to account for the presence of the solid ground, the ground reflection factor k_r is introduced[7], the ideal value of which is 2 (corresponding to a total reflection) and typical value 1.9 (corresponding to a standard terrain).

Randall's approach[5] is employed to derive the ray-tracing equations for the trajectory of the acoustic rays, while for the variation of the ray-tube area the expression derived from [6] and simplified as proposed in [3] is adopted.

The nonlinear distortion of the pressure signature is caused by the difference between the actual time t_r spent by each portion of the acoustic perturbation to reach the ground according to its actual propagation speed, which is the sum of the local sound speed a and the flow perturbation velocity w, and the ideal time t_i predicted by classic linear acoustics, to travel the same distance. In this boom propagation model, the pressure signature is supposed to be defined in time as $\Delta P = \Delta P(t, r)$ at any distance r from the aircraft's x axis in the vertical plane; nevertheless, it can trivially be defined also in space as $\Delta P = \Delta P(x, r)$ by considering that $x = Ut$ in steady flight, U being the flight speed. Considering the quasi-linear perturbation (3) the model by George and Plotkin[3] is recovered, whereas by substituting the ground intensity of the acoustic perturbation as calculated by (4) the nonlinear time advancement results:

$$\Delta t = -\frac{\gamma+1}{2\gamma}\left(\frac{\Delta P}{P}\right)_h \int_{s_h}^{s} \left[\frac{\sqrt{\frac{P_h a_h A_h}{P_0 a_0 A}}}{1+\frac{\gamma+1}{2\gamma}\sqrt{\frac{P_h a_h^2}{a_h s_h}}\left|\left(\frac{\Delta P}{P}\right)_h\right| \int_{s_h}^{s} \sqrt{\frac{a_0 A_h}{P_0 A}}\frac{ds}{a_0^2}} \right] \frac{ds}{a_0}, \quad (5)$$

where $ds = M_0 dr/\beta_0$, $M_0 = U/a_0$ and $\beta_0 = \sqrt{M_0^2 - 1}$. When the integral in the above formula converges to a finite number we observe the so-called "freezing effect"[8]. Typically, the ground pressure results in a non-physically multi-valued signature and the "area rule" needs to be employed in order to make it single-valued by inserting the appropriate shock waves[4], the area underneath the pressure signature being balanced on both sides of each shock.

3. RESULTS

As an example of the sonic boom propagation in the real atmosphere, we consider the pressure signature from a low-boom optimized Northrop F-5E Tiger II military aircraft[9] which is $L = 50$ft long and flies at $M = 1.4$ speed and $Z_f = 32000$ft altitude.

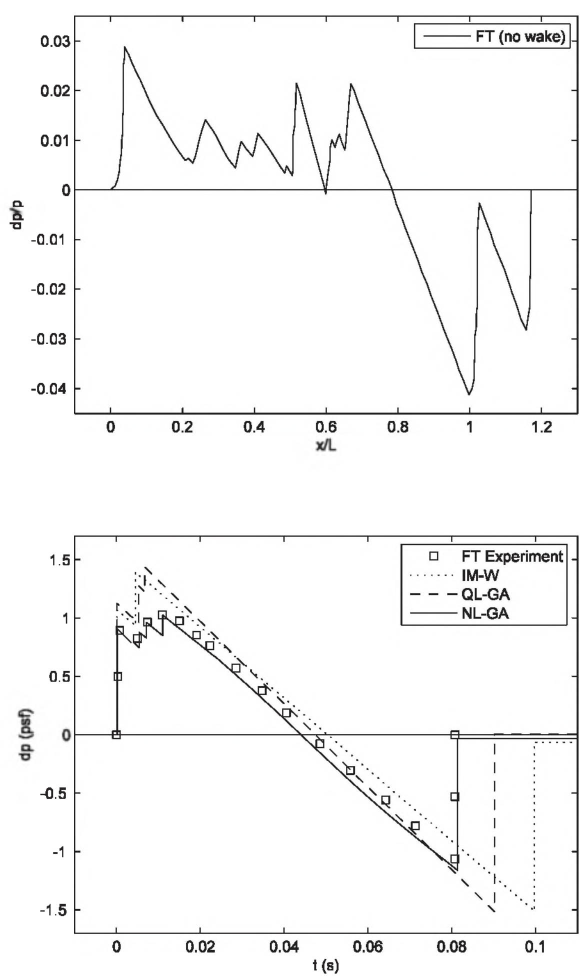

Figure 1. Initial and ground pressure signatures (wake removed)

The initial pressure signature ΔP_h is measured at a distance $r_h = 2L$ from the aircraft's trajectory during a flight test, by employing a follower aircraft. Along with the initial pressure signature, results for the ground pressure signature are shown in Figure 1 and compared with that originally obtained directly from de-turbulenced ground measurements during the flight test. The initial pressure signature has been modified by neglecting the aircraft wake contribution. For this case the ground pressure signatures calculated by the quasi-linear model does not fully agree with the measured one, in terms of amplitude of the front shock system and extension; on the contrary, the ground pressure signature calculated by the nonlinear geometrical acoustic model agrees well with the measured one.

4. DISCUSSION

Further comparisons with previously published results, obtained with well established propagation codes, show that the differences in the ground pressure signatures predicted by quasi-linear and nonlinear propagation models increase dramatically with increasing the shock intensity and the aircraft speed, due to strongly nonlinear flow in the near-field, and with decreasing the aircraft length and increasing the flight altitude, due to cumulative effect of the small nonlinearities of the flow in the far-field over a long propagation distance. Good consistency and poor sensitivity of the propagation models to the initial conditions was also found. Although simple, the proposed nonlinear method allows a very efficient and accurate prediction of the boom propagation starting from a given pressure signal in the near-field and can therefore be considered a useful tool for the aerodynamic design and multi-objective optimization of low-boom supersonic aircrafts via CFD methods.

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