1. INTRODUCTION

Turbulent boundary layer (TBL) is a major source of aircraft cabin interior noise. In fact, jet powered aircraft cabin interior noise is mostly generated by the external flow excitation and engine noise. However, while during takeoff the engine is the dominant source of noise, in cruise flight the airflow sources are the major contribution for the interior noise [1]. As referred in [2], TBL excitation is regarded as the most important noise source for jet powered aircraft at cruise speed, particularly, as new quieter jet engines are being developed. For these reasons, reducing the turbulent flow induced noise in aircraft cabin is an important topic of research. Still, since the TBL is a stochastic phenomenon and due to the complexity of the aircraft structure itself, this is an ongoing topic of investigation. In order to successfully design effective noise control systems, a clear understanding of the mechanisms involved in the aircraft cylindrical cabin TBL-induced noise, such as the sound transmission and radiation of the coupled structural-acoustic system, is crucial.

In this context, the main goal of the present paper presents an analytical framework, developed by the authors, for the prediction TBL-induced noise into a cylindrical cabin, and its validation against experimental studies. To validate the analytical framework, several studies were considered for comparison. The acoustic enclosure is of cylindrical shape, filled with air, with simply supported end caps, and with a flexible cylindrical shell. The flexible cylindrical shell is backed by random noise or by turbulent flow. Furthermore, the model was extended for the cylindrical structure divided by several curved panel. The closed-form analytical solution of the coupled response of the system is obtained, and the analytical expressions were derived. As concluded in [3-5], both the structural and acoustic resonances have a significant contribution for the interior noise in the cabin. For this reason, the dynamic response of a fully coupled vibro-acoustic system is derived in terms of the acoustic modes and structural modes. It is shown that the analytical model provides a good prediction of the reality, and that it is important to consider the coupling between structural and acoustic systems. Analytical predictions are obtained for random and TBL excitations, both for the shell vibration and sound pressure levels.

2. ANALYTICAL FRAMEWORK

2.1 TBL Model for Cylindrical Coordinates

As described in more detail in references [3, 5], the power spectral density (PSD) of the TBL wall pressure fluctuations over a flat panel, \( p_{TBL}(x, y, t) \), can be defined using Corcos model by

\[
S(x, \xi_y, \omega) = S_{ref}(x, \omega) e^{-\frac{k c}{U c} \xi_x} e^{-\frac{k c}{U c} \xi_y},
\]

in which \( \xi_x = x - x' \) and \( \xi_y = y - y' \) are the spatial separations in the \( x \)- and \( y \)-directions, \( S_{ref}(x, \omega) \) is the reference PSD, \( U_c \) is the TBL convective speed. Assuming that, in substitution of a flat panel one has a shallow open circular shell, equation (1) can be written in the cylindrical coordinates systems as

\[
S(x, \xi_x, \xi_y, \omega) = S_{ref}(x, \omega) e^{-\frac{k c}{U c} \xi_x} e^{-\frac{k c}{U c} \xi_y} e^{-\frac{\omega \xi_y}{U c}},
\]

where \( \xi_y = (\theta - \theta') \) and \( R \) is the radius of the cylinder.

2.2 Cylindrical Shell Structural Model

For a given applied external pressure, \( p_{ext}(x, \theta, t) \), the shell governing equation may be defined as follows

\[
w + \frac{1}{V} + \frac{1}{E_s h_s} + v \frac{\partial^2 w}{\partial \theta^2} + \frac{h_s^2}{12R^2} \frac{\partial^2 w}{\partial \theta^2} - \frac{1}{E_s h_s} w = \frac{(1 - V^2) R^2}{E_s h_s} p_{ext}(x, \theta, t),
\]

in which \( V_c \) is the Poisson ratio of the shell, \( h_s \) is the shell thickness, \( E_s \) is the shell Young modulus, and \( \zeta_s \) was added to account for the damping of the shell. The structural displacement may be defined as follows

\[
w(x, \theta, t) = \sum_{m_x=1}^{M_x} \sum_{m_y=1}^{M_y} \alpha_{m_x}(x) \beta_{m_y}(\theta) q_{m_x m_y}(t),
\]

in which \( \alpha_{m_x}(x) \) and \( \beta_{m_y}(\theta) \) are the spatial functions, and \( q_{m_x m_y}(t) \) is a function of time. For simply supported circular cylindrical shells without axial constraint at its ends, the spatial functions are defined by

\[
\alpha_{m_x}(x) = \frac{2}{L_y} \sin \left( \frac{m_x \pi x}{L_y} \right), \quad \beta_{m_y}(\theta) = \frac{1}{\sqrt{\pi}} \cos \left( m_y \theta \right),
\]

where \( L_y \) is the cabin length, while for a open curved panel with simply supported edges, the spatial functions are
\[
\alpha_{nm}(x) = \frac{1}{\sqrt{a}} \sin\left(\frac{m_0 \pi (x-x_i)}{a}\right), \quad \beta_{nm}(0) = \frac{1}{\sqrt{\theta_0}} \sin\left(\frac{m_0 \pi (0-\theta_i)}{\theta_0}\right),
\]

in which \(a\) is the shell length, \(\theta_0\) is its arc, \(x_i\) and \(\theta_i\) are the starting coordinates of the shell in the cabin global coordinates system.

### 2.3 Cylindrical Cabin Acoustic Model

The governing equation of cylindrical acoustic enclosure is the wave equation written in the cylindrical coordinates system, as

\[
\nabla^2 p - \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} - \xi_{ac} \frac{\partial p}{\partial t} = 0,
\]

where \(c_0\) is the speed of sound in the enclosure, and the damping term \(\xi_{ac}\) was added to account for the acoustic damping, and \(p\) is defined by

\[
p(x, \theta, r, t) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{\eta=0}^{\infty} \psi_{nm}(x) \phi_{n\eta}(\theta) \Gamma_{n\eta}(r) r_{n\eta\eta}(t),
\]

in which \(r_{n\eta\eta}(t)\) are functions of time, \(\psi_{nm}(x)\), \(\phi_{n\eta}(\theta)\) and \(\Gamma_{n\eta}(r)\) are the spatial functions, defined as follows

\[
\psi_{nm}(x) = \frac{A_{nm}}{\sqrt{L_n}} \cos\left(\frac{n \pi x}{L_n}\right), \quad \phi_{n\eta}(\theta) = \frac{1}{\sqrt{\pi}} \sin\left(\eta \theta + \frac{\pi}{2}\right),
\]

\[
\Gamma_{n\eta}(r) = \frac{A_{n\eta}}{R} J_n\left(\frac{\lambda_{n\eta\eta} r}{R}\right),
\]

where \(J_n\) are Bessel functions of the \(n\)th kind, \(\lambda_{n\eta\eta}\) factor represents the \(n\)th root of \(d J_n(\lambda_{n\eta\eta} r)/dr = 0\), at \(r=R\).

### 2.4 Structural-Acoustic Coupled Model

Following a similar mathematical procedure as the one described in [3, 5], the governing equations for the coupled system are obtained from the combination of the governing equations of the individual systems, and applying the boundary conditions, which in the present study are

\[
\frac{\partial p(x, \theta, r, t)}{\partial r} = \begin{cases} \rho_0 \tilde{W}(x, \theta, t), & \text{at } r = R \\ 0, & \text{elsewhere}. \end{cases}
\]

### 3. RESULTS AND DISCUSSION

Two independent studies were used as validation cases for the analytical framework. In this paper only one of the cases is presented. The study in [6] presents a numerical formulation based on a variational approach, in order to investigate the vibroacoustic behavior of a cylindrical shell, and the effect of an internal floor partition. The numerical results are also compared with experimental results. Figure 1 (b) displays the results from [6], showing the acoustic effect of the floor on the internal pressure field. Figure 1 (a) shows the SPL results obtained through our analytical framework. To obtain the analytical results, a total number of \(M_x=15\) and \(M_0=15\) shell modes, and \(N_x=10\), \(N_0=10\) and \(N_\eta=10\) acoustic modes were used to achieve convergence. Comparing the analytical predictions in part (a) with the results in part (b) (solution without attached floor), it can be observed that the developed analytical framework is in good agreement with the data from [6].

![Fig. 1. SPL results: (a) obtained with our analytical framework; (b) from [6]: with attached floor, without attached floor.](image)

### REFERENCES