ANALYSIS OF SPATIAL RESONANCE IN A SMALL VESSEL TO STUDY VIBRATION-INDUCED DIGITAL VASCULAR DISORDER

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1. INTRODUCTION

Numerous construction, forest workers and miners are exposed to hand-arm vibration from tools. Prolonged exposures to hand-vibration have been identified to cause hand-arm vibration syndrome (HAVS). HAVS is a collective term, which consists of disorders of a musculoskeletal, vascular and neurological nature. But the exact pathophysiology is still unknown. One of the HAVS components is Vibration White Finger (VWF) that causes severe blanching due to loss of blood, followed by episodic and painful return of blood circulation.

A number of experimental studies performed to understand the relationship between the damage in the arterial wall and vibration (Bovenzi et al., 2006), but most of these studies are non-invasive or on various animals like rat-tail or rabbit. According to Curry et al., (2005), the ultrastructural appearance of the internal elastic membrane (IEM) was most modified by 800 Hz vibration and the pattern of damage above 800 Hz suggests that the IEM is destroyed because it resonates and absorbs vibration energy at this frequency. In this paper, the vibration response of small arteries is studied by modeling the system as an elastic tube filled with incompressible fluid embedded in an elastic foundation.

2. METHODS AND RESULTS

The blood flow through the artery is represented in Figure 1.

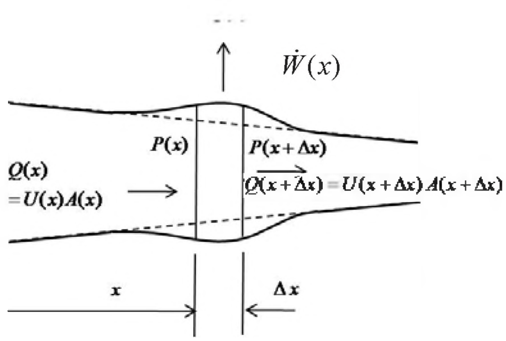

Figure 1. Schematic of flow through artery

Using the volume velocity Q(x)=U(x) A(x), where U(x) is particle velocity and W(x) is wall distension:

$$Q(x)-Q(x+\Delta x)=\dot{W}(x)2\pi R(x)\Delta x \cong P(x)Y_w(x)\Delta x$$

$$P(x)-P(x+\Delta x)=Z_f(x)\Delta x Q(x)$$

$Y_w(x)$ is the admittance of the artery wall and $Z_f(x)$ the impedance of the flow. They are defined as:

$$Y_w=\left(j\omega L_w+R_w+\frac{1}{j\omega C_w}\right)^{-1},\quad Z_f(x)=\frac{j\omega\rho_f}{A(x)} \text{ where}$$

$$L_w=\frac{\phi\rho_w h}{2\pi R(x)},\ R_w=\frac{r_w}{2\pi R(x)},\ \frac{1}{C_w}=\frac{Eh}{2\pi R^3(x)}$$

We assume that the radius changes linearly. The equations can be represented by the circuit diagram shown in Figure 3.

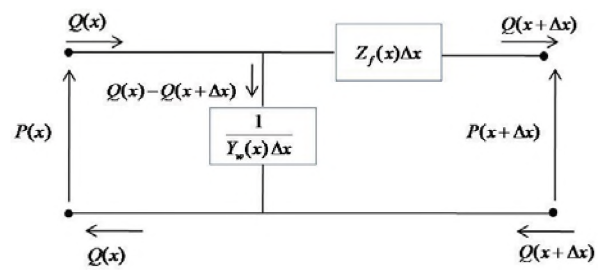

Figure 2. Equivalent Circuit for a small section of artery.

The entire artery system can be represented by a series of circuits shown in Figure 2, as it is shown in Figure 3. In the figure, $Z_{ni}=1/Y_{ni}(x)$ is the structural impedance of i[th] section of the artery wall. Looking at Figure 3 and the system equations, it is recognized that the wave problem in an artery of varying diameter is represented by exactly the same equations as those of waves in the cochlea (Zweig, 1976). The cochlea is a wave guide that has fluid-structure interaction, which is comprised of the basilar membrane whose admittance varies as a function of the distance from the oval window and is loaded by the cochlea fluid. The derivation in this paper follows the work by Zweig on the motion of the cochlea in response to a sound input.

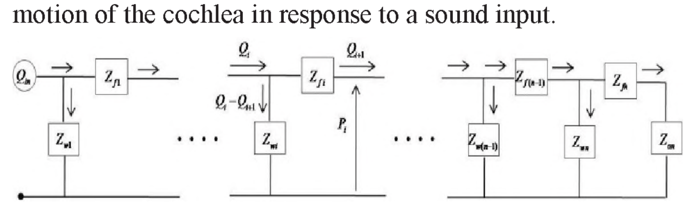

Figure 3. Long artery with linearly changing radius.

2.1. Local Resonance Frequency

As can be seen in Figure 3, segments of the artery system can be considered to be small oscillatory dynamic systems connected serially by the fluid impedance.

Therefore, it is expected that the vibration input (Q) coming from the left end will keep leaking through the circuit branch with Z_{ni}. The "resonance frequency" $\omega_r(x)$ and damping ratio $\delta(x)$ of each segment of the system are –

$$\omega_r(x) = \left(L_w C_w\right)^{-1/2}, \ \delta(x) = \omega_r(x) R_w(x) C_w$$

These properties are functions of the axial position. The resonance frequency of the digital artery is calculated with 0.1mm thickness, 0.49 Poisson's ratio, 15 MPa Young's modulus and 50mm length of the vessel (Kuwabara et al., 2008; Langewouters et al., 1986). The diameter linearly increases from 0.52 mm to 1.06mm over the length of 50mm. Figure 4 shows the resonance frequency calculated as a function of the length of the vessel. The resonance frequency decreases with distance from the left end.

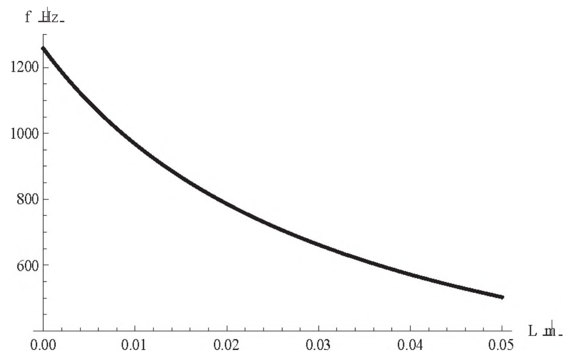

Figure 4. Resonance frequencies in Hz versus length in m.

2.2. Spatial Resonance Phenomenon

When the cochlea are subjected to sound of a given frequency, the basilar membrane vibrates with large amplitude at a particular position, and the position depends on the frequency of the sound. This is called spatial resonance (Zweig, 1976), which explains how frequency can be sensed by the ear. Because the equation of motion of the artery system is the same as that of the cochlea, the spatial resonance phenomenon is expected also in the artery system. Figure 5 shows the displacement amplitude at two different frequencies with input from smaller and larger diameter sides. A very interesting observation is that when the input is from the large diameter side, the same artery system does not show any spatial resonance. This phenomenon can be explained qualitatively by the circuit diagram shown in Figure 3. When the input is from the large diameter side, artery impedances are small in the input side. Therefore, the flow (current) through these impedances is large in the beginning. Thus the vibration energy flowing downstream decreases very quickly, making only a very small amount of the flow reach the resonance location.

3. DISCUSSIONS AND CONCLUSION

It has been shown that the frequency of local resonance of an artery whose diameter changes is a function of the axial position. The equations of motion as well as the circuit description of the motion are recognized as the same as those of the motion of the cochlea. As it is expected from the similarity with the cochlea, spatial resonance is observed, however only when the vibration input is from the small diameter (stiffer) side. It is noted that cochlea input is always from the stiffer side.

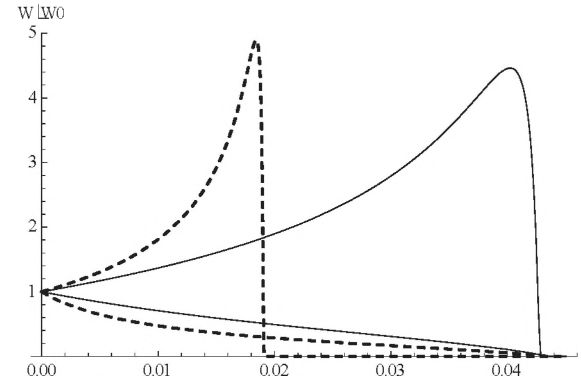

Figure 5. Displacement of the artery wall as a function of the axial distance in meters. Solid: 550 Hz, dashed: 800 Hz. Curves with peaks are when the input is from the small diameter side and curves that decrease without any peak are when the input is from the large diameter side.

This spatial resonance phenomenon may explain the pathophysiology of the circulation system disorder that causes white finger disease. If a worker uses a specific type of tool for a prolonged period, he/she will be subjected to vibration of high amplitudes with the same dominant frequency components. Therefore, the finger will be subjected to vibration of large amplitude always at same locations, which will cause hardening of the artery wall and surrounding tissue. Further work is underway to verify the behaviour at various other frequencies using bench-top tests and other equivalent models.

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