COCHLEAR EVANESCENT LIQUID SOUND-PRESSURE WAVES DURING SPONTANEOUS OTO-ACOUSTIC EMISSIONS

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1. INTRODUCTION

In the diagram on the left in Fig. 1, the streamlines of an evanescent (standing) liquid sound-pressure wave generated by a miniaturized and idealized underwater tuning-fork prong are shown [Frosch (2010a, 2010b)]. The prong is assumed to oscillate in the z_r-direction. Liquid particles having a no-wave location on one of these streamlines stay on that line during their oscillation. In the diagram on the right in Fig. 1, the corresponding lines of constant liquid sound-pressure amplitude are displayed; see Section 2.1.

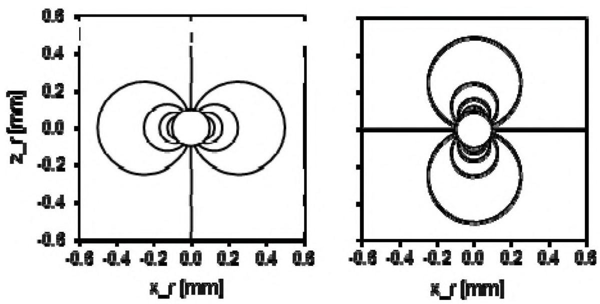

Figure 1. Underwater mini-tuning-fork prong oscillating in the z_r-direction. Left: streamlines. Right: lines of constant liquid sound-pressure amplitude.

In this study it is assumed that spontaneous oto-acoustic emissions (SOAEs) from the human inner ear [e.g., Frosch (2010a)] are generated, in a feedback process, by outer-hair-cell-driven localized oscillations of the basilar membrane (BM), and it is shown that a corresponding liquid motion above and below the BM of an idealized cochlear box model [cubic channel, x-independent properties] can be found by a superposition of three standing waves similar to that shown in Fig. 1, generated by a prong centred at $x_r = 0$ and by two prongs at $x_r = \pm a$, where typically $a = 0.01$ mm. It is assumed that at time $t = T/4$ (where T = oscillation period) the central prong is at $z_r = +2\delta$ and the two lateral prongs are at $z_r = -\delta$; typically, $\delta = 0.1$ µm.

2. METHODS AND RESULTS

The liquid-particle oscillation amplitudes are small compared to the half-pressure distance of ~0.1 mm visible in Fig. 1. In the corresponding small-displacement approximation [see e.g. Frosch (2010a)] the sound-pressure p and the liquid-particle velocity v in a liquid of density ρ and of negligible compressibility and viscosity obey Newton's second law in the form

$$\rho \cdot (\partial \vec{v} / \partial t) = -\vec{\nabla} p \, , \qquad (1)$$

and, in the present two-dimensional case, the Laplace equation,

$$\partial^2 p / \partial x^2 + \partial^2 p / \partial y^2 = 0 \, . \qquad (2)$$

A possible (standing-wave) solution for p is:

$$p(x, y, t) = a_p(x, y) \cdot \sin(\omega \cdot t) \, , \qquad (3)$$

where the angular frequency $\omega = 2\pi \cdot f$ is assumed to be constant. The real function $a_p(x, y)$ in Eq. (3) must fulfil the Laplace equation (2).

2.1. Oscillation of a "miniprong" according to Fig. 1

As explained e.g. in Frosch (2010b), the real and imaginary parts of the function $F(n_c) = n_c^{-1}$ (where $n_c = x_r + i \cdot z_r$) yield the circular streamlines and lines of constant sound-pressure amplitude shown in Fig. 1. Streamlines:

$$r = R \cdot (N / n) \cdot \cos(\varphi) \; ; \qquad (4)$$

$n = 0, \pm1, \pm2, \ldots, \pm N$ is the running number of the streamlines; in Fig. 1 $N = 5$ was chosen; $R = 0.1$ mm is the prong radius; r and φ are plane polar coordinates:

$$r = \sqrt{x_r^{\,2} + z_r^{\,2}} \, ; \; \tan(\varphi) = z_r / x_r \, . \qquad (5)$$

Lines of constant sound-pressure amplitude:

$$r = R \cdot (a_{p0} / a_p) \cdot \sin(\varphi) \; . \qquad (6)$$

In Eq. (6), a_{p0} is a pressure constant, and a_p is defined by Eq. (3). The lines of constant liquid sound-pressure amplitude in Fig. 1 are for $a_p / a_{p0} = 0.0, \pm0.2, \pm0.4, \ldots, \pm1.0$.

2.2. Localized oscillation of the basilar membrane

The superposition described in Section 1 yields the following liquid sound-pressure amplitude [Frosch (2011)]:

$$\frac{a_p}{a_{p0}} = \frac{-R \cdot z_r}{x_r^{\,2} + z_r^{\,2}} + \frac{0.5R \cdot z_r}{(x_r - a)^2 + z_r^{\,2}} + \frac{0.5R \cdot z_r}{(x_r + a)^2 + z_r^{\,2}} ; \qquad (7)$$

The corresponding streamlines are defined as follows:

$$\frac{q \cdot n}{N} = \frac{R \cdot x_r}{x_r^{\,2} + z_r^{\,2}} - \frac{0.5R \cdot (x_r - a)}{(x_r - a)^2 + z_r^{\,2}} - \frac{0.5R \cdot (x_r + a)}{(x_r + a)^2 + z_r^{\,2}} ; \qquad (8)$$

the number q in Eq. (8) is the maximum of the expression on the right-hand side for $z_r = R$; for $R = 0.1$ mm and $a = 0.01$ mm one finds $q = 0.007224$; that maximum is located at $x_r = 41.7 \mu\text{m}$; see Fig. 3.

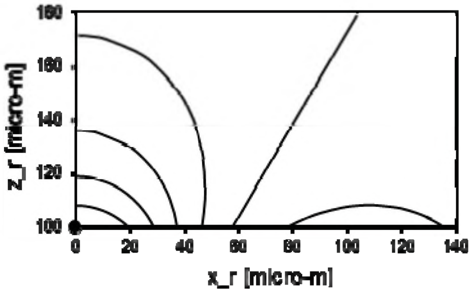

Figure 2. Constant-pressure lines, $a_p/a_{p\min}$ = 0.8, 0.6, 0.4, 0.2, 0.0, –0.2, according to Eq. (7); $a_{p\min}$ is the value of a_p at $x_r = 0$, $z_r = R = 0.1$ mm; the BM is at $z_r = R$.

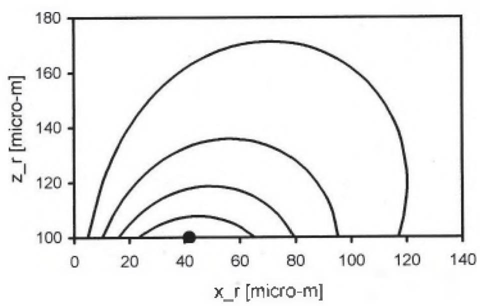

Figure 3. Streamlines according to Eq. (8), for $N = 5$.

The BM is assumed to have a negligible thickness and to be located at $z_r = R = 0.1\,\mathrm{mm}$ In the case of both Fig. 2 and Fig. 3, the patterns at $z_r < R$ (below the BM) and at $x_r < 0$ are mirror images of those shown.

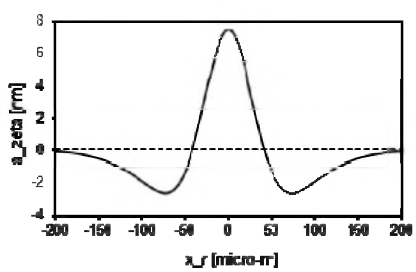

Figure 4. Shape of the basilar membrane at time $t = T/4$, according to Eqs. (1,3,7); $z_r = R = 100$ µm; $a_{p0} = 1$ Pa; oscillation frequency $f = 1$ kHz; liquid density 1 g/cm^3.

The amplitudes of the z-components $\zeta = a_\zeta \cdot \sin(\omega t)$ of the displacements of the liquid particles in Figs. 2 and 3 from their no-wave locations x_r, z_r, according to Eqs. (1), (3), and (7), are given by Eq. (25) of Frosch (2011). The corresponding shape of the BM at time $t = T/4$ is shown in Fig. 4.

3. DISCUSSION AND CONCLUSIONS

The evanescent (standing) liquid sound-pressure wave described in Section 2.2 fulfils the Laplace equation. It is however incompatible with Newton's second law (force = mass × acceleration) applied to the friction-less passive basilar-membrane (BM) elements of a cochlear box model [Frosch (2010a)] with x-independent BM stiffness S and BM surface mass density M, and with negligible direct mechanical coupling of the BM elements. That disagreement can be corrected by introduction of a periodic force exerted on the BM by active outer hair cells (OHCs). In the present frictionless case, the OHC force must be proportional to $\sin(\omega t)$. In Fig. 5, the amplitude of the required force on a BM element of $5 \times 300\,\mu\mathrm{m}^2$, centred at x_r, is shown versus x_r for three different values of the assumed BM stiffness.

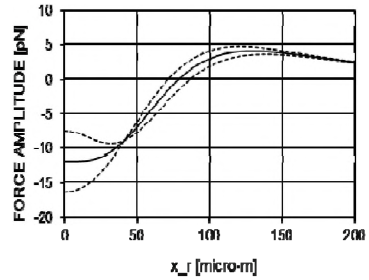

Figure 5. Active-outer-hair-cell force amplitude required by Newton's law (see text). Curves starting at –16.4, –12.0, and –7.6 pN are for S / S_{res} = 1.3, 1.4, and 1.5.

The quantity S_{res} in the caption of Fig. 5 is the without-liquid BM resonance stiffness, $S_{res} = \omega^2 \cdot M$; e.g., if $S = 1.4 S_{res}$ (solid curve in Fig. 5), then for the parameters in the caption of Fig. 4 and for $M = 0.1\,\mathrm{kg/m^2}$, $t = T/4$, $x_r = 0$, the forces due to the BM stiffness, the liquid-pressure difference across the BM, and the active OHCs amount to –62.0, +29.7, and –12.0 pN. The resultant force, –44.3 pN, agrees with the downwards acceleration of the considered BM element. If both the liquid and the OHC force were removed, then the BM free-oscillation frequency in that box model with stiffness $S = 1.4 S_{res}$ would be $\sqrt{1.4} \cdot 1\,\mathrm{kHz}$ = 1.18 kHz, higher than 1 kHz by about 0.24 octave.

In the real cochlea, "slow" travelling surface waves of given frequency [Frosch (2010a)] are impossible at the without-liquid BM resonance place for that frequency, but are possible at the corresponding with-liquid resonance place, which is more basal by typically 0.24 octave distance, i.e., by about 1.1 mm. From that place, the with-liquid BM oscillations conjectured to occur during spontaneous oto-acoustic emissions could be efficiently carried, by such slow cochlear travelling waves, to the cochlear base (oval window, stapes) and then propagate to the ear canal.

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