# Acoustic Response of Multi-Fluid Shell Systems with Structural Enhancement 

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## 1. INTRODUCTION

The present study is a further development of our earlier work on a shell with a co-axial core but identical internal and external fluids [1,2], and a shell filled with and submerged into two different fluids [3], both subjected to an external shock wave. Specifically, it was shown that when the internal and external fluids are different, there are up to four different scenarios of the interaction possible, depending on the properties of the fluids. It was also shown that placing a rigid core inside a shell filled with and submerged into identical fluids dramatically changes both internal and external acoustic patterns. Thus, investigating the result of combining these two enhancements of the system's complexity is of definite interest.

## 2. MATHEMATICAL FORMULATION AND SOLUTION METHODOLOGY

We consider a thin elastic circular cylindrical shell of radius $\mathrm{r}_{0}$ filled with and submerged into different fluids, and containing a rigid co-axial core of radius a. We assume that the shell is thin, and that its deflections are small in comparison to its thickness, so that the linear theory of shells can be employed; we further assume that the LoveKirchhoff hypothesis holds true (using the Reissner-Mindlin model was shown to provide more accurate results in the present context [4], but the accuracy provided by the LoveKirchhoff model is adequate for our purposes). The fluids are assumed to be irrotational, inviscid, and linearly compressible.

The methodology developed in our earlier work [58] is used, i.e. we apply the Laplace transform time-wise and use the separation of variables for the spatial coordinates to obtain the pressure in terms of the series with time-dependent coefficients. The approach has been extensively validated and the results it produces have shown excellent agreement with experiments.

## 3. RESULTS AND CONCLUSIONS

A steel shell is considered with the thickness of 0.01 m and radius of 1 m , submerged into and filled with fluids of the same density but with varying acoustic speeds, and containing a rigid co-axial core, also of varying radius. The interaction with a cylindrical acoustic pulse is analyzed. and it is assumed that the rate of the exponential decay behind the front is 0.0001314 s and the peak pressure in the front is 250 kPa .

The "classical" case of the identical fluids inside and outside the shell was addressed in detail in [1], and the profound effect that the core, especially of the large radius, has on the hydrodynamic fields was discussed (the respective images are not reproduced here). We focus on determining what is the effect of the core on the hydrodynamic fields when the fluids are different.

Fig. 1 and 2 show the hydrodynamic field induced by the pulse on a shell with three different cores, a small-, medium-, and large-radius ones for the scenario where the internal acoustic speed is higher than external (or $\zeta>1$; in this case, $\zeta \sim 1.43$ where $\zeta$ is defined as the ratio of the internal and external acoustic speeds [3]). The instant at which the fields were simulated was chosen to represent one of the most important stages of the interaction when the post-reflection focusing is developing in the internal fluid [3]. We can see that the focusing pattern characteristic of this stage is present for the small and, largely, medium core but is absent for the large core. Furthermore, the entire sequence of the internal pressure wave propagation, reflection, and focusing is "shifted" in time which results in the "head" waves seen in front of the incident wavefront.


Figure 1. The hydrodynamic pattern for $\zeta \sim 1.43$ and two different cores, $\mathrm{a} / \mathrm{r}_{0}=0.10$ (top image) and $\mathrm{a} / \mathrm{r}_{0}=0.50$ (bottom image); $\mathbf{t = 1 . 7 0}$.

Fig. 3 shows the hydrodynamic fields for the same shell and two cores, a small-radius one and a medium-radius one, but for a different value of $\zeta$, that is, for the case where the internal acoustic speed is lower than the external one ( $\zeta \sim 0.57$ ), the scenario that was found to be most interesting in our earlier work[3]. We can see that, even for a smallradius core, the overall hydrodynamic field has a much
more complex structure than in the absence of the core [3]: the same is definitely true for the medium-size core. This complexity requires a more in-depth study and is the subject of our current investigation.


Figure 2. The hydrodynamic pattern for $\zeta \sim 1.43$ and $a / r_{0}=0.75$; $\mathrm{t}=1.70$.


Figure 3. The hydrodynamic pattern for $\zeta \sim 0.57$ and two different cores, $a / r_{0}=0.10$ (top image) and $a / r_{0}=0.50$ (bottom image); $\mathbf{t = 3 . 7 0}$.

We conclude that, very much like in the case of the identical fluids [3], the presence of the core dramatically changes the phenomenology of the interaction, and leads to a complete disappearance of some of the phenomena that
are observed in the absence of the core. However, unlike in the case of the identical fluids, the diversity of the interaction is now incomparably greater for the hydrodynamic patterns observed change not only with the radius of the core but also with the acoustic properties of the fluids.

The scenario of $\zeta<1$, very much like in the case when no core is present, appears to be most interesting, and definitely deserves a more in-depth investigation. It is expected that the study of the diversity of the wave propagation, reflection, and focusing it that case will reveal some very interesting and, possibly, practically consequential effects that are not observed when the core is present but the fluids are identical or when the fluids are different but the core is not present.

## REFERENCES

[1] Iakovlev, S., Gaudet, J., Dooley, G., MacDonald, B. (2010) Hydrodynamic fields induced by the shock response of a fluidfilled submerged cylindrical shell containing a rigid co-axial core, Journal of Sound and Vibration 329 (16), 3359-3381.
[2] Iakovlev, S. (2004) Influence of a rigid coaxial core on the stress-strain state of a submerged fluid-filled cylindrical shell subjected to a shock wave, Journal of Fluids and Structures 19 (7), 957-984.
[3] Iakovlev, S. (2009) Interaction between an external shock wave and a cylindrical shell filled with and submerged into different fluids, Journal of Sound and Vibration 322 (1-2), 401-437.
[4] Iakovlev, S., Santos, H. A. F. A., Williston, K., Murray, R., Mitchell, M. (2011) Non-stationary radiation by a cylindrical shell: numerical modeling using the Reissner-Mindlin theory, submitted to Journal of Fluids and Structures.
[5] Iakovlev, S. (2006) External shock loading on a submerged fluid-filled cylindrical shell, Journal of Fluids and Structures 22, 997-1028.
[6] Iakovlev, S. (2007) Submerged fluid-filled cylindrical shell subjected to a shock wave: Fluid-structure interaction effects, Journal of Fluids and Structures 23 (1), 117-142.
[7] Iakovlev, S. (2008) Interaction between a submerged evacuated cylindrical shell and a shock wave. Part I: Diffraction-radiation analysis, Journal of Fluids and Structures 24, 1077-1097.
[8] Iakovlev, S. (2008) Interaction between a submerged evacuated cylindrical shell and a shock wave. Part II: Numerical aspects of the solution, Journal of Fluids and Structures 24 (7), 1098-1119.

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