

ANALYSIS OF THE EXTREME ACOUSTIC PRESSURE IN MULTI-FLUID SHELL SYSTEMS SUBJECTED TO AN EXTERNAL PULSE

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1. INTRODUCTION

As was recently established, the interaction between an external shock wave or acoustic pulse and a cylindrical shell filled with and submerged into different fluids is very complex, with up to four different scenarios possible [1]. This complexity was analyzed in some detail in that paper, but not all its aspects were covered.

Specifically, two important issues were not addressed. The first is the actual evolution of the hydrodynamic fields as the transition between different interaction scenarios happens – it is of definite interest to see exactly how the structure of the fields changes. The second, more practically important, issue is the analysis of the peak pressure that is observed as a result of the many shock wave reflection, propagation and focusing phenomena that take place during the evolution in question.

Our goal, therefore, is to present a summary of the most important aspects of the evolution of the hydrodynamic fields, and to complement that with an assessment of the maximum pressure observed in the system.

2. MATHEMATICAL FORMULATION AND SOLUTION METHODOLOGY

We consider a thin elastic circular cylindrical shell filled with and submerged into different fluids. We assume that the shell is thin enough, and that its deflections are small in comparison to its thickness, so that the linear shell theory can be employed; we further assume that the Love-Kirchhoff hypothesis holds true. We note that although using the Reissner-Mindlin model was shown to provide more accurate results [2], employing the Love-Kirchhoff model is still more than acceptable for the purposes of the present study.

The fluids are assumed to be irrotational, inviscid, and linearly compressible, thus the wave equations are used to model the fluid dynamics. The fluids and the shell are coupled through the dynamic boundary condition on the interface.

As it was established [1], a single most important parameter determines the appearance of the hydrodynamic fields observed in the system, namely ζ which is defined as the ratio of the sound speed in the internal fluid to that in the external one. Changing ζ implies varying the acoustic properties of the fluids.

The problem is approached with the methodology developed in our earlier work [3-6], i.e. we apply the Laplace transform time-wise to the wave equations and then separate the spatial variables in order to arrive at the expressions for the transforms of the internal and external pressure in a form of a series of modified Bessel functions of the first (internal fluid) and second (external fluid) kind.

The pressure is then obtained as a Fourier series with time-dependant coefficients which, for the radiation pressure, depend on the unknown normal displacements of the shell.

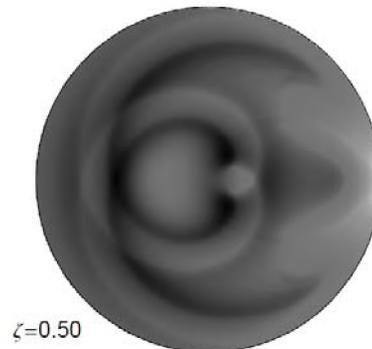
Then, the same series form is used for the shell displacements and, substituting them into the shell equations, we arrive at the systems of the ordinary differential equations for each of the displacement harmonics. The systems are then approached numerically (finite differences) and the resulting normal displacement is used to compute the radiation pressure.

We note that although the present approach is used, in this case, to model structurally simple system (a single shell), it can also be successfully employed to address more complex structures [7].

3. RESULTS AND CONCLUSIONS

A steel shell is considered with the thickness of 0.01 m and radius of 1 m, submerged into and filled with fluids of the same density but the acoustic speeds varying according to the changes of ζ . The interaction with a cylindrical pulse is analyzed, and the rate of the exponential decay is assumed to be 0.0001314 s while the initial pressure in the front is 250 kPa.

Fig. 1 shows three numerically simulated images of the internal field for the chosen pulse for various ζ during the reflection of the internal pressure wave off the surface of the shell which is one of the most important stages of the process. The evolution of the respective hydrodynamic pattern is clearly visible, and different values of ζ yield very different fields. Of particular interest to us here is the point-like high-pressure region observed right at the shell surface at $\zeta=0.50$ as it is expected to yield a peak of internal hydrodynamic pressure.



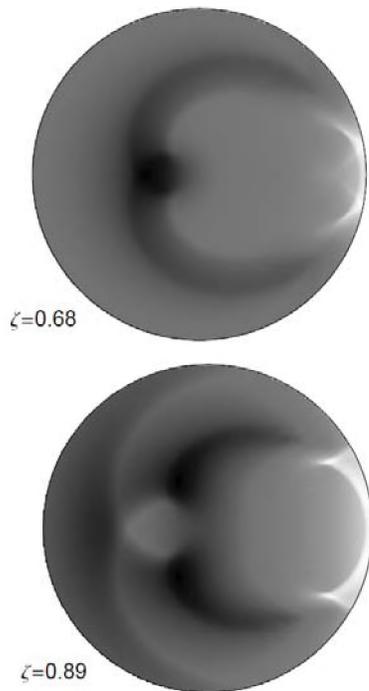


Figure 1. The internal hydrodynamic pattern for three different values of ζ .

For the other two values of ζ , the reflection pattern is less localized, and is expected to yield lower pressure than the $\zeta=0.50$ case.

In order to assess the extremities of the hydrodynamic field, we analyze the highest pressure induced by the four phenomena: the two focusings (the pre- and post-reflection) and the reflection [1], and the peak pressure associated with the Mach stems [3], and plot it versus ζ for its practically meaningful values, Fig. 2.

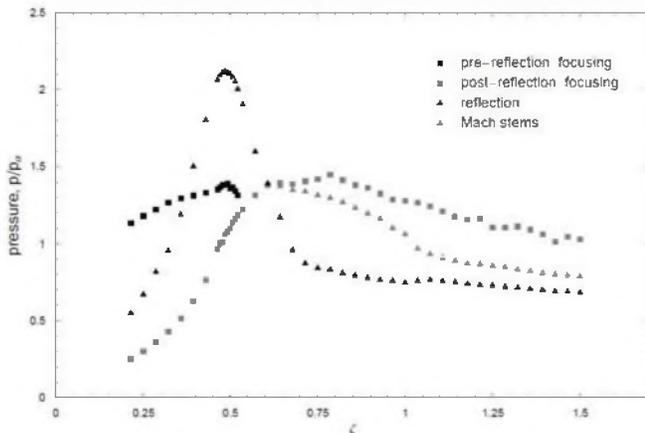


Figure 2. The maximum pressure inside the shell for various ζ .

As one can see, the overall highest pressure is associated with the reflection and is achieved at $\zeta \sim 0.48$. This corresponds to the transition between the scenario where both pre- and post-reflection focusing are observed to the scenario where only one, post-reflection focusing takes place (the latter being characteristic of the “classical”

scenario of identical internal and external fluids). This peak pressure exceeds the peak incident pressure by 110%, a result that has obvious and far-reaching practical consequences. From the phenomenological point of view, the peak pressure in question is originated as a result of the near-simultaneous occurrence of the pre-reflection focusing and reflection at or very near the shell surface (note the termination of the pre-reflection focusing shortly after that).

For higher values of ζ , the peak pressure is due to the post-reflection focusing, a phenomena that was discussed in the present context in [3]. We also note that any value of ζ in the interval $[0.38, 0.58]$ yields very high pressure that exceeds the peak incident one by at least 50%. The highest pre- and post-reflection focusing pressure and the highest pressure associated with the Mach stems exceed the peak incident pressure by 40%, 45%, and 40%, respectively.

The observations made underscore the necessity for a careful pre-design analysis of multi-fluid shell systems in the context of impulse loading, especially when there is at least some freedom in varying the properties of the fluids. Even when the properties cannot be changed, the analysis in question is still valuable as to ensure that the extreme pressure observed during the interaction is accounted for.

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