BAYESIAN LOCALIZATION OF MULTIPLE OCEAN ACOUSTIC SOURCES WITH ENVIRONMENTAL UNCERTAINTIES

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1. INTRODUCTION

This paper considers simultaneous localization of multiple acoustic sources when properties of the ocean environment (water column and seabed) are poorly known [1, 2]. A Bayesian formulation is applied in which the environmental parameters, noise statistics, and locations and complex strengths (amplitudes and phases) of multiple sources are considered unknown random variables constrained by acoustic data and prior information. The posterior probability density (PPD) over all parameters is defined and integrated using efficient Markov-chain Monte Carlo methods to produce joint marginal probability densities for source ranges and depth. This approach also provides quantitative uncertainty analysis for all parameters, which can aid in understanding the inverse problem and may be of practical interest (e.g., source-strength probability distributions). Closed-form maximum-likelihood expressions for source strengths and noise variance at each frequency (developed in the following section) allow these parameters to be sampled implicitly, substantially reducing the dimensionality and difficulty of the inversion. An example is presented of multiple-source localization in an uncertain shallow-water environment.

2. THEORY

Consider measured data $\mathbf{d} = \{\mathbf{d}_f; f = 1, N_F\}$ consisting of complex acoustic fields at N_F frequencies recorded at an array of N_H hydrophones. The acoustic field at each frequency is assumed to be due to N_S sources at locations $\mathbf{x} = \{\mathbf{x}_s = (r_s, z_s); s = 1, N_S\}$, where r_s and z_s are the range and depth of the sth source. The complex source strengths (amplitude and phase) are denoted $\mathbf{a} = \{[\mathbf{a}_f]_s\}$. The data errors are considered complex Gaussian-distributed random variables with unknown standard deviation at each frequency denoted $\boldsymbol{\sigma} = \{\sigma_f\}$. The unknown environmental parameters are denoted by $\mathbf{e}$. Under these assumptions, the likelihood function is given by [2]

$$L(\mathbf{x},\mathbf{e},\mathbf{a},\boldsymbol{\sigma};\mathbf{d}) = \prod_{f=1}^{N_F} \frac{1}{(\pi\sigma_f^2)^{N_H}} \times \exp\Big\{-\Big|\,\mathbf{d}_f - \sum_{s=1}^{Ns}[\mathbf{a}_f]_s \mathbf{d}_f(\mathbf{x}_s,\mathbf{e})\Big|^2 / \sigma_f^2\Big\}, \quad (1)$$

where $\mathbf{d}_f(\mathbf{x}_s,\mathbf{e})$ represent the modelled acoustic field for a unit-amplitude, zero-phase source at location $\mathbf{x}_s$ in an environment $\mathbf{e}$. Rearranging, the data misfit (negative log-likelihood) function is given by

$$E(\mathbf{x},\mathbf{e},\mathbf{a},\boldsymbol{\sigma};\mathbf{d}) = \sum_{f=1}^{N_F} \big|\,\mathbf{d}_f - \mathbf{D}_f\mathbf{a}_f\,\big|^2 / \sigma_f^2 + 2N_H \log_e \sigma_f, \quad (2)$$

where the complex matrix $\mathbf{D}_f$ is given by

$$[\mathbf{D}_f]_{hs} = [\mathbf{d}_f]_h(\mathbf{x}_s,\mathbf{e}). \quad (3)$$

To estimate ML source strengths, setting $\partial E/\partial \mathbf{a}_f = 0$ for Eq. (2) leads to

$$\hat{\mathbf{a}}_f = (\mathbf{D}_f^H\mathbf{D}_f)^{-1}\mathbf{D}_f^H\mathbf{d}_f, \quad (4)$$

where H indicates Hermitian (conjugate) transpose. Substituting this estimate back into the original misfit function (2) leads to a new misfit

$$E(\mathbf{x},\mathbf{e},\boldsymbol{\sigma};\mathbf{d}) = \sum_{f=1}^{N_F}\Big|\big[\mathbf{I}-\mathbf{D}_f(\mathbf{D}_f^H\mathbf{D}_f)^{-1}\mathbf{D}_f^H\big]\mathbf{d}_f\Big|^2/\sigma_f^2 + 2N_H\log_e\sigma_f. \quad (5)$$

To estimate ML standard deviations, $\partial E/\partial \sigma_f = 0$ leads to

$$\hat{\sigma}_f^2 = \frac{1}{N_H}\big[\mathbf{I}-\mathbf{D}_f(\mathbf{D}_f^H\mathbf{D}_f)^{-1}\mathbf{D}_f^H\big]\mathbf{d}_f. \quad (6)$$

Substituting this estimate back into misfit function (5) and neglecting additive constants (representing fixed normalization factors for the likelihood) leads to new misfit

$$E(\mathbf{x},\mathbf{e};\mathbf{d}) = N_H\sum_{f=1}^{N_F}\log_e\Big|\big[\mathbf{I}-\mathbf{D}_f(\mathbf{D}_f^H\mathbf{D}_f)^{-1}\mathbf{D}_f^H\big]\mathbf{d}_f\Big|^2. \quad (7)$$

Evaluating misfit function (7) for a specific source location and environment ($\mathbf{x}$ and $\mathbf{e}$) implicitly applies the ML estimates for complex source strengths and standard deviations ($\mathbf{a}$ and $\boldsymbol{\sigma}$). This implicit sampling can replace explicit sampling in optimization and integration algorithms reducing the dimensionality of the inversion from $2N_SN_F + N_F+2N_S+N_E$ to $2N_S+N_E$, where N_E is the number of unknown environmental parameters. For instance, in the example presented in the following section the dimensionality is reduced from 35 to 14. Finally, it is interesting to note that in the special case of a single source ($N_S = 1$), the magnitude-squared term of Eq. (7) reduces to

$$\Big|\big[\mathbf{I}-\mathbf{D}_f(\mathbf{D}_f^H\mathbf{D}_f)^{-1}\mathbf{D}_f^H\big]\mathbf{d}_f\Big|^2 = \big|\mathbf{d}_f\big|^2 - \frac{\big|\mathbf{d}_f^H(\mathbf{x},\mathbf{e})\,\mathbf{d}_f\big|^2}{\big|\mathbf{d}_f(\mathbf{x},\mathbf{e})\big|^2}, \quad (8)$$

which is equivalent to the Bartlett misfit commonly used in matched-field localization and geoacoustic inversion.

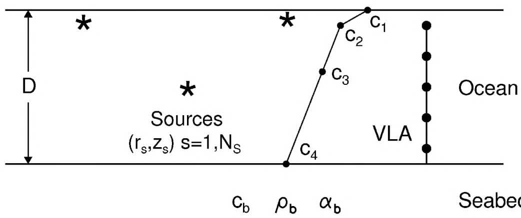

Figure 1. Schematic diagram of the geometry of the three-source localization indicating unknown environmental parameters.

3. EXAMPLE

The multiple-source localization procedure outlined above is demonstrated here with a synthetic example [2], as illustrated in Fig. 1. The geoacoustic parameters include the sound speed c_b, density ρ_b, and attenuation α_b of a uniform seabed. The water-column sound speed profile is represented by four unknown sound speeds c_1-c_4 at depths of 0, 10, 50, and D m, where D is the water depth. All environmental parameters are considered unknown with prior information consisting of uniform distributions over wide bounds. The three acoustic sources are located at (r, z) = (7 km, 4 m), (3 km, 2 m) and (5.4 km, 50 m) with relative amplitudes of 1, 0.5 and 0.13, phases of 45°, 90° and −90°, and signal-to-noise ratios of 10, 4 and −4 dB, respectively, at three frequencies of 300, 400 and 500 Hz. Simulated acoustic data were computed for a 24-sensor vertical line array using a normal mode propagation model. Fig. 2 shows joint marginal probability densities over source range and depth: All sources are successfully localized, with the greatest uncertainty for the weak submerged source for which the marginal density is strongly multi-modal.

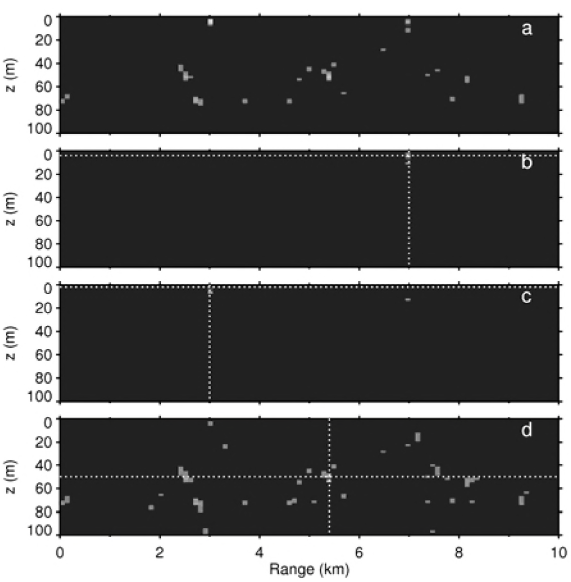

Figure 2. Joint marginal probability densities over source range and depth for all 3 sources (a), and individual sources (b-d). Dotted lines indicate true ranges and depths.

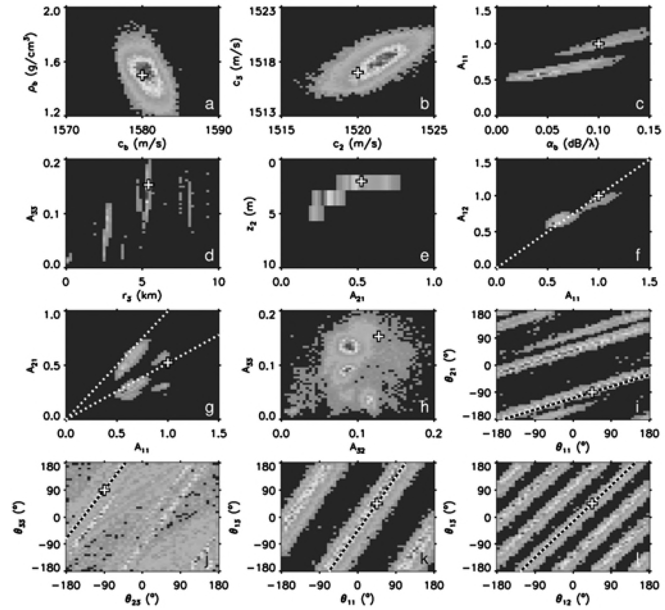

Figure 3. Joint marginal densities for selected geoacoustic parameters and source amplitudes and phases. Crosses indicate true values.

Fig. 3 illustrates joint marginal densities over selected environmental parameters and source amplitudes, A_{ij}, and phases, θ_{ij}, (where i indicates the source number and j the frequency number). Several interesting features can be observed. For instance, Fig. 3(f) shows that the probability for the amplitude of source 1 at frequencies 1 and 2 is concentrated along the (dotted) line $A_{11} = A_{12}$, which represents the correct inter-frequency scaling. Fig. 3(g) shows that the highest probability for the amplitude of sources 1 and 2 at frequency 1 follows the correct scaling $A_{21} = 0.5A_{11}$ (lower dotted line), and virtually all probability satisfies $A_{21} < A_{11}$ (i.e., is below the upper dotted line). Amplitude relationships for the weak source 3 are less clearly defined (Fig. 3h). Fig. 3(i) and 3(j) show that the joint marginal probability for the phase of different sources at the same frequency follow a (dotted) line with slope equal to the ratio of the source ranges (with phase wraps), as can be derived from modal considerations [2]. Finally, Fig. 3(k) and 3(l) show that the joint marginal probability for the phase of the same sources at different frequencies follow a (dotted) line with slope equal to the ratio of the frequencies, which can also be derived from modal considerations [2].

REFERENCES

[1] Dosso, S.E. and M. J. Wilmut, 2009. Comparison of focalization and marginalization for Bayesian tracking in an uncertain ocean environment, J. Acoust. Soc. Am., **125**, 717-722.

[2] Dosso, S.E. and M. J. Wilmut, 2011. Bayesian multiple-source localization in an uncertain environment, J. Acoust. Soc. Am., **129**, 3577-3589.