BAYESIAN TRACKING OF MULTIPLE OCEAN ACOUSTIC SOURCES WITH ENVIRONMENTAL UNCERTAINTIES

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1. OVERVIEW

This paper describes a Bayesian approach to the problem of simultaneous tracking of multiple acoustic sources in a shallow-water environment in which water-column and seabed properties are not well known [1, 2]. The Bayesian formulation is based on treating the environmental parameters, noise statistics, and locations and complex strengths (amplitudes and phases) of multiple sources as unknown random variables constrained by acoustic data formulated in terms of a likelihood function and be prior information (bounds on source speed and environmental parameters). Markov-chain Monte Carlo methods are applied to numerically sample the posterior probability density (PPD) to integrate over unknown environmental parameters in a principal-component space. Closed form maximum-likelihood expressions for source strengths and noise variance at each frequency allow these parameters to be sampled implicitly, substantially reducing the dimensionality of the inversion [3]. The result is a set of time-ordered joint marginal probability distributions for the range and depth of each source, which quantify the information content for multiple-source tracking. Optimal track estimates (with uncertainties) can be extracted from the marginal densities using the Viterbi algorithm [4].

2. EXAMPLE

The multiple-source tracking procedure outlined in the previous section is demonstrated here with a two-source synthetic example involving shallow and deep sources moving along similar tracks. The parameters of the ocean environment are illustrated in Fig. 1. The geoacoustic parameters include the thickness h of an upper sediment layer with sound speed c_s, density ρ_s, and attenuation α_s, overlying a semi-infinite basement with sound speed c_b, density ρ_b and attenuation α_b. The water-column sound speed profile is represented by four unknown sound speeds c_1-c_4 at depths of 0, 10, 50, and D m, where D is the water depth. All of these environmental parameters are considered to be unknown random variables with prior information consisting of uniform distributions over wide bounds representing physically reasonable limits (true parameter values and prior bounds are given in Table 1).

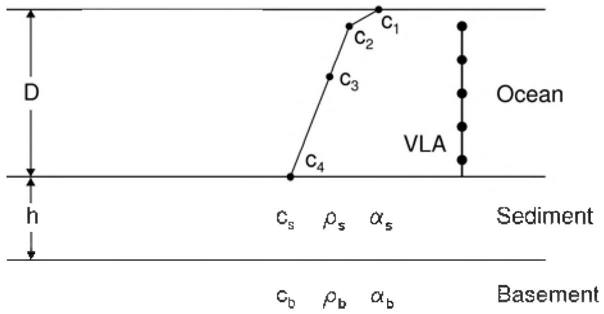

Figure 1. **Schematic diagram of the geometry of the two-source tracking problem, indicating unknown environmental parameters.**

Table 1. **True values and uniform prior bound limits for environmental parameters of the synthetic test case.**

Parameter & Units	Value	Prior Bounds
SSP:		
D (m)	100	[98, 102]
c_1 (m/s) @ 0 m	1520	[1516, 1524]
c_2 (m/s) @ 10 m	1514	[1510, 1518]
c_3 (m/s) @ 50 m	1509	[1505, 1513]
c_4 (m/s) @ 130 m	1507	[1505, 1513]
Seabed:		
h (m)	10	[0, 30]
c_s (m/s)	1500	[1450, 1600]
c_b (m/s)	1560	[1500, 1650]
ρ_s (g/cm^3)	1.45	[1.4, 2.2]
ρ_b (g/cm^3)	1.8-	[1.4, 2.2]
α_s (dB/λ)	0.05	[0, 1]
α_b (dB/λ)	0.10	[0, 1]

In the simulation, 300-Hz acoustic fields from the two moving sources are recorded at 120-s intervals for 16 minutes (i.e., 9 recordings) at a 24-sensor vertical line array (VLA) which spans the 100-m water column. (The acoustic fields are computed using a normal-mode propagation model with additive Gaussian noise.) The shallow source moves with an inward radial velocity of 7 m/s (14 kts) from 14- to 7.7-km range over the 16-minute time interval and remains at a constant depth of 4 m. The deep source moves at 8.5 m/s (17 kts) from 15- to 7.4-km range and decreases depth at a uniform rate from 80 to 64 m (a vertical velocity of 0.017 m/s). The signal-to-noise ratio (SNR) for the incoming deep source varies from 15-21 dB along the track; for the shallow source the SNR varies from 6-13 dB. In the

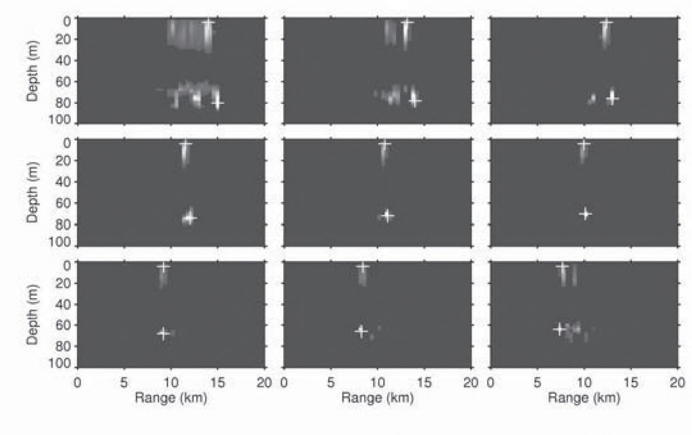

Figure 2. Joint marginal probability densities for the two-source tracking problem (time increases from left to right, top to bottom). Crosses indicate the true track position.

tracking inversion, the radial and vertical velocities are constrained to be less than 10 m/s and 0.0333 m/s, respectively.

Figure 2 shows the time-ordered sequence of marginal probability densities for range and depth for the two sources along the track. The two sources are generally well tracked, with range and depth uncertainties that decrease with source range (increasing SNR). At the longest ranges the marginal densities for both sources are multi-modal in range; however, the modes coalesce with decreasing range.

Figure 3 shows the optimal source location estimates along the tracks extracted from the marginal densities via the Viterbi algorithm (applying the source velocity constraints noted above). One-standard deviation uncertainties in range and depth are included as error bars. The estimated track is close to the true track for both deep and shallow sources. The largest uncertainties occur at the longest ranges, and at the track endpoints due to the fact that only one-sided velocity constraints are applied here. The mean absolute range and depth errors are 41 m and 8 m respectively for the shallow source and 18 m and 1.1 m for the deep source.

3. SUMMARY

This paper presented a Bayesian approach to simultaneous tracking of multiple sources in an uncertain environment. Markov-chain Monte Carlo methods are applied to integrate the PPD over environmental nuisance parameters, yielding joint marginal probability densities for source ranges and depths, from which optimal source location estimates and uncertainties can be extracted. The approach was illustrated with a synthetic example of tracking a deep submerged source in the presence of a loud interfering surface source.

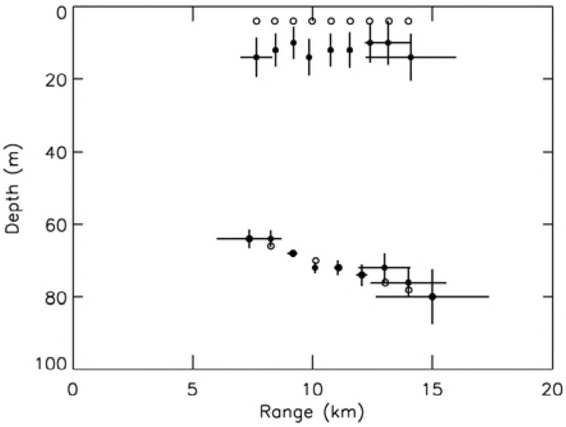

Figure 3. Optimal source locations for the two sources computed using the Viterbi algorithm (filled circles), with one-standard deviation error bars in range and depth. True source locations are indicated by open circles.

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