# Application of Modal Assurance Criterion on Metallic and Composite Structures

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## 1. INTRODUCTION

This paper presents the use of the Modal Assurance Criterion (MAC) for the purpose of spatially comparing mode shapes to identify differences in the degrees of freedom between test and analysis modes. In modal analysis, direct use of experimental results may include errors due to measurements limitations, for example modes duplications. Before using experimental data for analytical operations, it is essential to validate first experimental test results through correlation with FEM models [1]. The MAC methodology can be used to check this correlation. This article begins with an overview of methods used in test and analysis for correlation Next, MAC is evaluated using simple metallic beams and plates in various mounting (boundary) conditions. Finally, further experimental investigations using composite sandwich-composite panels are presented to demonstrate the practicality of this algorithm for real life applications.

### 1.1. Test and analysis using correlation [1]

Most popular applications of modal testing provide direct comparisons of deformed modal shapes between analytical and experimental findings. The responses of modal testing are measured at a number of sensors, which allow visualization of the measured motion. Visualization is key for a proper quality assessment of an experimental result. To do this effectively, several steps must be followed.

A. Topology correlation [2]:

Analysis performed using the Finite Element method gives predictions at DOFs {q} of the FEM model. FEM predictions and measurements {y} are not directly comparable. Thus the first step of correlation, called topology correlation, consists in building a function allowing prediction of FEM responses at sensors. In most applications, DOFs and responses are linearly related with an observation matrix [c], so that an observation equation can be written in the form [2]:

$$\{y(t)\} = [c]\,\{q(t)\}$$

B. Correlating shapes known at sensors

Before performing any operation to assess the quality of the analytical model, it is paramount that a correspondence be established between the analytical and experimental methods. The approximation of modal frequencies is sufficient evidence to do this, but a comparison of deformed modal shapes is recommended for further validation between analytical and experimental findings. The Modal Assurance Criterion can be used in this goal.

Modal Assurance Criterion (MAC):

A quantifiable correlation between experimental and analytical mode shapes can be determined based on the Modal Assurance Criteria (MAC) [3]. For two shapes U, V defined on the same sensors, the MAC is the correlation coefficient between the two vectors.

$$\mathrm{MAC}(U,V) = \frac{\left|\{U\}^T\{V\}\right|^2}{\left|\left(\{U\}^T\{U\}\right)\left(\{V\}^T\{V\}\right)\right|}$$

A perfect correlation between two modes gives a MAC result equal to 100%. An example definition of MAC correlations is provided in Table.1 below:

**Table 1: MAC correlations**

| | |
|---|---|
| 100 %≤ MAC≤ 90% | Correlated modes |
| 90 %≤ MAC≤ 70% | Doubtful correlation |
| 10 %≤ MAC≤ 70% | Uncorrelated modes |
| MAC=10% | Modes are nearly orthogonal |

## 2. MEASUREMENT SETUP

In Figure 1, the measurement setup to measure mode shapes and frequency response functions (FRFs) of simple metallic beams, plates and sandwich-composite panels in various mounting (boundary) conditions using a Laser Doppler Vibrometer (LDV - Polytec PDV-100) is shown. A shaker (B&K type 4810) was used for vibration excitation. In the experiment an initial “FFT” acquisition was performed to obtain FRFs using a periodic white noise excitation. This provided natural frequency determination by finding local maxima in an averaged FRF graph. Responses were measured at a number of sensors, which had a spatial distribution allowing a visualization of the measured motion

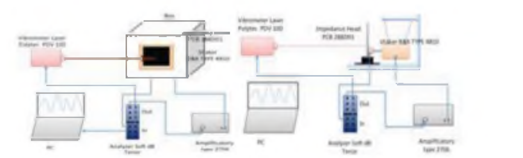


**Figure 1: Measurement Setup for measuring mode shapes and frequency response functions (FRFs) of simple metallic beams, plates, and sandwich-composite panel in various boundary conditions.**

## 3. RESULTS

An aluminum beam has been tested in clamped free condition by the setup mentioned above. Figure 2 presents the device for testing the aluminum beam.

**Figure 2: bench tests**

The MAC was applied. Figure 3 presents the Modal assurance criterion of the aluminum beam.

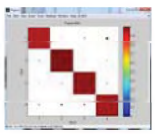
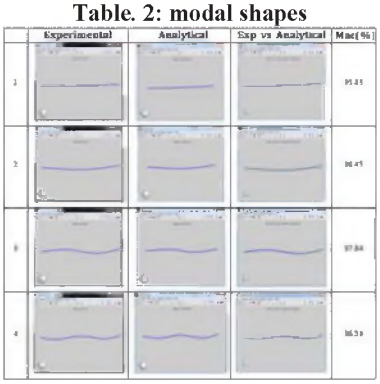


**Figure 3: Modal assurance criterion of the aluminum beam**

Table 3 presents a comparison between the modal shapes of the first four flexible modes, calculated and measured by SDtools.

**Table. 2: modal shapes**




A steel plate was also tested in both free-free and clamped conditions using the setup mentioned above.
Figure 4 presents the device for measuring the steel plate

**Figure 4: Bench tests**

Table 4 presents the MAC value between the modal shapes of the first four flexible modes, calculated and measured by SDtools

**Table 4: MAC Value**

| MAC (%) Free Free | MAC (%) Clamped |
|---|---|
| 92.05 | 99.22 |
| 92.97 | 86.61 |
| 88.84 | 85.68 |
| 85.54 | 91.81 |

Finally a composite-sandwich panel was tested with clamped boundary conditions using the same experimental setup as mentioned above. Figure 4 presents the device for measuring the composite sandwich panel. Figure 5 shows the frequency response functions (FRFs) of the sandwich-composite panel identified by SDtools.

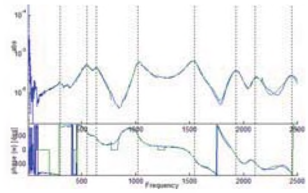
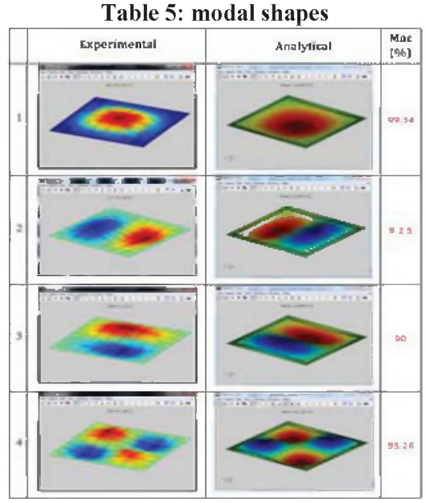


**Figure 5: Frequency response functions (FRFs) of the sandwich-composite panel**

Figure 5 shows a parasite frequency at 350Hz and 500Hz. These frequencies can be caused by the effect of the interaction between the cabin and the plate. To ensure the identified modes, MAC was used as a verification technique. Table 5 presents a comparison between the modal shapes of the first four flexible modes of the plate reference EDEC using analytical (Artec) and experimental (SDTools) techniques.

**Table 5: modal shapes**




## 4. DISCUSSION AND CONCLUSIONS

The MAC methodology was evaluated on simple metallic beams, plates and sandwich-composite panels. MAC was verified on real measured data, which lead to the necessity of this algorithm for real life applications especially for sandwich-composite structures. Finally MAC overcame some of the limitations of experimental measurement such as mode duplication.

## REFERENCES

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