# NONLINEAR GEOACOUSTIC INVERSION VIA PARALLEL TEMPERING

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## 1. INTRODUCTION

Knowledge of seabed geoacoustic and scattering properties is important for sonar, geophysical, and geotechnical applications in shallow-water environments. Since direct measurements can be time consuming and expensive, inferring *in-situ* information about seabed model parameters from the inversion of ocean acoustic data has received a great deal of attention. A Bayesian approach to geoacoustic inversion [1,2] provides quantitative uncertainty analysis and has been applied for a variety of acoustic data types, including the inversion of full-field, reflection, dispersion, and reverberation measurements.

In Bayesian inversion, unknown model parameters are considered random variables constrained by measured data and prior information, with the goal of estimating integral properties of the multi-dimensional posterior probability density (PPD), such as marginal probability distributions. For nonlinear problems, such as geoacoustic inversion, numerical methods must be applied to estimate these integrals. In particular, the Markov-chain Monte Carlo method of Metropolis-Hastings sampling (MHS) has been applied in virtually all Bayesian geoacoustic inversions to date [1]. However, MHS can be inefficient for strongly nonlinear inverse problems involving PPDs with multiple modes (i.e., multiple isolated regions of high probability in the parameter space). For such problems MHS has the potential to miss important regions of the parameter space and to significantly under-estimate parameter uncertainties. Multi-modal PPDs have been observed for all of the geoacoustic data types mentioned above.

This paper applies the method of parallel tempering [3,4] to achieve efficient and effective sampling of a particularly challenging multi-modal problem involving the inversion of acoustic reverberation data for geoacoustic and scattering parameters. Parallel tempering has the ability to transition freely between multiple PPD modes by running parallel Markov chains at a series of increasing sampling temperatures $T$, with probabilistic interchanges between chains. High-$T$ chains provide wider sampling of the parameter space and the possibility of bridging isolated modes, while low-$T$ chains provide more precise local sampling but are prone to become trapped in localized regions of the space. Parallel tempering improves sampling by providing interchange between chains at different temperatures. Including higher-$T$ chains ensures that the lower-$T$ chains can access all regions of the space while still providing efficient local sampling, resulting in a robust ensemble sampler.

## 2. EXAMPLE

This section compares MHS and parallel tempering for Bayesian geoacoustic inversion of simulated (noisy) reverberation data. Simulation provides a number of advantages for such comparisons, in that an appropriate model parameterization is known and the error statistics are also known and controlled. Hence, characteristics of the inversion, such as PPD multi-modality, arise solely from the physics of the forward problem, and are not an artifact of a poor choice or parameterization or unaccounted-for sources of error.

The (range-independent) seabed model assumed for the reverberation inversion problem is illustrated Fig. 1. The seabed is represented by an upper sediment layer of thickness $h$=5 m, sound velocity $v_1$=1470 m/s, density $\rho_1$=1.4 g/cm$^3$, and attenuation $\alpha_1$=0.5 dB/wavelength, overlying a semi-infinite basement with corresponding parameters $v_2$=1660 m/s, $\rho_2$=1.8 g/cm$^3$, and $\alpha_2$=0.1 dB/wavelength. Acoustic backscatter occurs at rough interfaces at the top and bottom of the sediment layer and at heterogeneities within the volume of the sediments. The spatial roughness of the upper and lower interfaces is assumed to be isotropic and characterized by a two-dimensional power-law spectrum $R_i(k) = w_i k^{-\gamma_i}$, where $k$ is the magnitude of the horizontal wave vector, $w_i$ is the spectral strength, and $\gamma_i$ is the spectral exponent, with $i$=1,2 corresponding to the upper and lower interfaces, respectively (values used here are $w_1 = w_2$= 0.02 and $\gamma_1 = \gamma_2$ = 3). The volume-scattering intensity cross-section for the sediments is given by $S_V$ = $10^{-6}$ m$^{-3}$. Finally, the standard deviation of the data errors, $\sigma$ =1 dB, is also considered an unknown parameter in the inversion. The reverberation data are shown in Fig. 2.

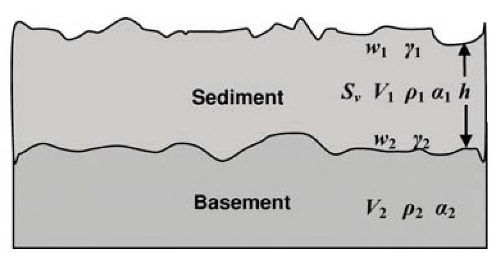


**Figure 1.** Schematic diagram of the two-layer seabed model indicating unknown parameters. The error standard deviation is also included as an unknown in the inversion.

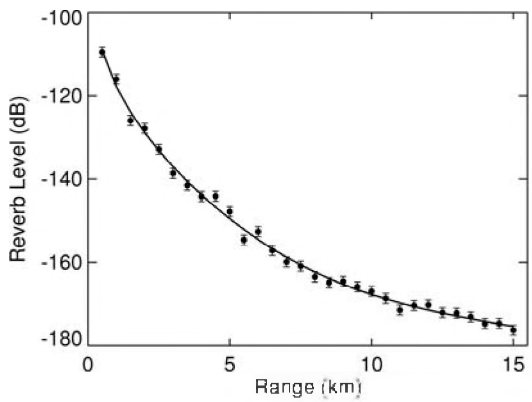


**Figure 2.** Simulated noisy reverberation data (circles) and modelled data (solid line) computed for the highest-probability model. Error bars indicate the maximum-likelihood standard-deviation Estimate evaluated at the most-probable model.

Both MHS and parallel tempering were applied to the reverberation inversion problem outlined above. The parallel tempering approach made use of a total 30 parallel chains, with 16 chains at $T = 1$, 8 chains at $T = 2$, 4 chains at $T = 4$, and 2 chains at $T = 8$. Results are based on the $T$=1 and 2 chains, with the samples collected at $T = 2$ re-weighted so as to remove the sampling bias which otherwise occurs for sampling at non-unity temperature [5].

All 13 model parameters described above were included in the two inversion approaches. However, given space constraints and to highlight sampling of multi-modal PPD structure, results are considered here only in terms of joint marginal probability distributions for sediment thickness and sound velocity which is highly multi-modal, as shown in Fig. 3. This figure compares inversion results for MHS and parallel tempering after various numbers of samples, as indicated on each panel (for parallel tempering the total number of samples is indicated, including samples at higher temperatures which were not included in the marginal distribution estimate).

Considering first the MHS results (left column of Fig. 3), over the first $10^5$ samples (top panel), the method has sampled only a single mode of the PPD, and if sampling was terminated here the multi-modality would go undetected and parameter uncertainties would be substantially under-estimated. A second PPD mode is detected by $2x10^5$ samples, and third mode by $5x10^5$ samples. The marginal distribution does not change significantly in going from $10^5$ to $10^6$ samples, but by $2x10^6$ samples additional modes are detected. From the behavior in Fig. 3, it appears unlikely that MHS has visited all modes even with $2x10^6$ samples, and it is clear the sampling is not even close to convergence (typically requiring many transitions between all modes).

Considering the parallel-tempering results (right column of Fig. 3), it is clear that the multi-modality of the joint marginals for $h$ and $v_1$ are mapped out far better with $10^5$ total samples using parallel tempering (top panel on right) than with $2x10^6$ samples using MHS (bottom panel on left). In particular, the parallel-tempering marginal distribution includes multi-modal structure which is not apparent with MHS for sample sizes up to 20 times larger. Further, the fact that there is little practical difference in results for different parallel-tempering sample sizes in Fig. 3 indicates convergence by $10^5$ samples, and hence large sample sizes are not required to map the complicated multi-modal PPD structure using this approach.

## REFERENCES

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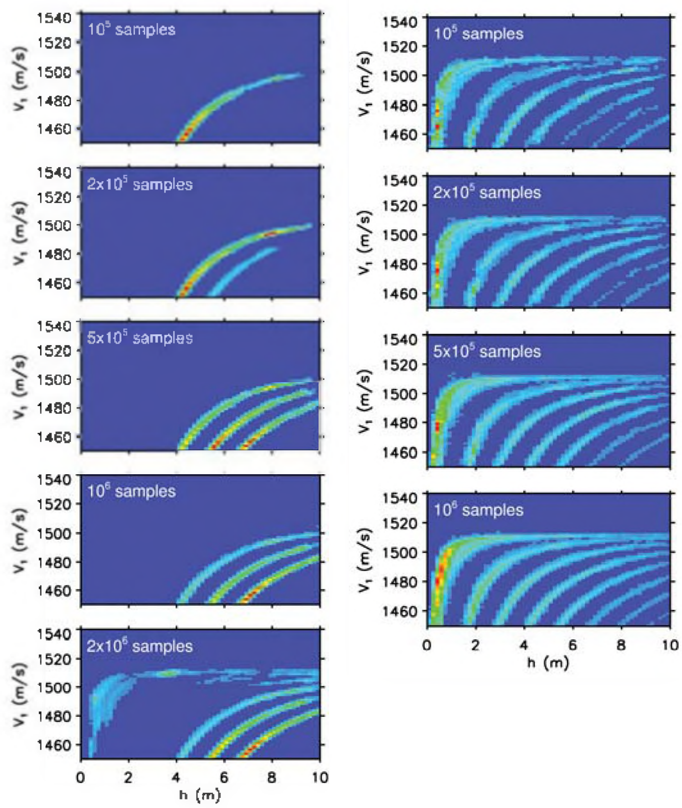


**Figure 3.** Comparison of joint marginal probability distributions for $h$ and $v_1$ as computed using MHS (left column) and parallel tempering (right column) using the number of samples indicated on each panel.