EFFICIENT BAYESIAN MULTI-SOURCE LOCALIZATION

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1. INTRODUCTION

This paper develops and illustrates an efficient approach to the simultaneous localization of an unknown number of ocean acoustic sources, based on minimizing the Bayesian information criterion (BIC) over source parameters [1]. A Bayesian formulation is developed in which the number, locations, and complex strengths (representing amplitudes and phases) of an unknown number of sources are considered random variables constrained by acoustic data and prior information. The BIC, which balances data misfit with a penalty for extraneous parameters, is minimized using simulated annealing, with Gibbs sampling applied over source locations. Closed-form maximum-likelihood (ML) expressions for source strength and noise variance at each frequency allow these parameters to be sampled implicitly, substantially reducing the dimensionality and improving the efficiency of the inversion. Gibbs sampling and the implicit formulation provide an efficient scheme for adding and deleting sources with a reasonable acceptance rate during the optimization. A simulated example is presented which considers localizing several quiet submerged sources in the presence of multiple loud near-surface interfers.

2. THEORY

This section develops the Bayesian approach to multiple-source localization. Consider data $\mathbf{d} = \{\mathbf{d}_f;\ f = 1, N_F\}$ consisting of complex (frequency-domain) acoustic fields at an array of N_H hydrophones for N_F frequencies. The field at each frequency is assumed to be due to $s = 1, N_S$ acoustic sources at locations (ranges and depths) $\mathbf{x} = \{\mathbf{x}_s,\ s = 1, N_S\} = \{(r_s, z_z)\ s = 1, N_S\}$ with complex strengths $\mathbf{a} = \{[a_f]_s\}$. Errors on $\mathbf{d}_f$ are assumed to be complex Gaussian distributed with unknown variance v_f. Let $\mathbf{m} = \{\mathbf{x}, \mathbf{a}, \boldsymbol{v}\}$ represent the set of unknown model parameters. Data and parameters are considered to be random variables related by Bayes' rule

$$P(\mathbf{m}\,|\,\mathbf{d}, N_S) = \frac{P(\mathbf{d}\,|\,\mathbf{m}, N_S)P(\mathbf{m}\,|\,N_S)}{P(\mathbf{d}\,|\,N_S)}. \tag{1}$$

In Eq. (1), the posterior probability density (PPD), $P(\mathbf{m}|\mathbf{d})$, represents the state of information for the parameters incorporating both data information, $P(\mathbf{d}|\mathbf{m})$, and prior information, $P(\mathbf{m})$. Interpreting the conditional probability $P(\mathbf{d}|\mathbf{m})$ as a function of $\mathbf{m}$ for the (fixed) observed data $\mathbf{d}$ defines the likelihood function $L(\mathbf{m}) \propto \exp[-E(\mathbf{m})]$, where E is the data misfit (log likelihood) function. Given the assumptions stated above, the likelihood is given by

$$L(\mathbf{x}, \mathbf{a}, \mathbf{v}) = \prod_{f=1}^{N_F} \frac{1}{(\pi \nu_f)^{N_f}} \exp\left\{ -\left| \mathbf{d}_f - \sum_{s=1}^{N_S} a_{fs} \mathbf{d}_f(\mathbf{x}_s) \right|^2 / \nu_f \right\}$$

$$= \frac{1}{\prod_{f=1}^{N_f} (\pi \nu_f)^{N_f}} \exp\left\{ -\sum_{f=1}^{N_F} \left| \mathbf{d}_f - \mathbf{D}_f \mathbf{a}_f \right|^2 / \nu_f \right\} \tag{2}$$

where $\mathbf{d}_f(\mathbf{x}_s)$ represents the modelled acoustic fields for a unit-amplitude, zero-phase source at location $\mathbf{x}_s$ and $\mathbf{D}_f$ is an N_H by N_S complex matrix of acoustic fields defined

$$[\mathbf{D}_f]_{hs} = [\mathbf{d}_f(\mathbf{x}_s)]_h. \tag{3}$$

Equation (2) can be written $L \propto \exp[-E]$ where the misfit function is given by

$$E(\mathbf{x}, \mathbf{a}, \mathbf{v}) = \sum_{f=1}^{N_F} \left(| \mathbf{d}_f - \mathbf{D}_f \mathbf{a}_f |^2 / \nu_f + N_H \log_e \nu_f \right). \tag{4}$$

Implicit sampling over source strengths and noise variances is derived by setting $\partial E / \partial \mathbf{a}_f = \partial E / \partial \nu_f = 0$ to obtain ML estimates

$$\hat{\mathbf{a}}_f = \left(\mathbf{D}_f^H \mathbf{D}_f \right)^{-1} \mathbf{D}_f^H \mathbf{d}_f$$
$$\hat{\mathbf{v}}_f = \frac{1}{N_H} \left| \left[\mathbf{I} - \left(\mathbf{D}_f^H \mathbf{D}_f \right)^{-1} \mathbf{D}_f^H \right] \mathbf{d}_f \right|^2 \tag{5}$$

where ^{H} indicates Hermitean (conjugate transpose) and $\mathbf{I}$ is the identity matrix. Applying these in Eq. (4), the misfit can be written

$$E(\mathbf{x}) = N_H \sum_{f=1}^{N_F} \log_e \left| \left[\mathbf{I} - \mathbf{D}_f (\mathbf{D}_f^H \mathbf{D}_f)^{-1} \mathbf{D}_f^H \right] \mathbf{d}_f \right|^2. \tag{6}$$

Evaluating Eq. (6) for specific $\mathbf{x}$ automatically applies the ML solution for $\mathbf{a}$ and $\mathbf{v}$, and hence accounts for the corresponding variability in source strengths and variances implicitly. This greatly reduces the dimensionality and improves the efficiency of multiple-source localization.

Determining the number of sources that contribute significantly to the acoustic field may be considered an application of model selection; i.e., seeking the most appropriate N_S given the measured data **d**. In Baye's rule, Eq. (1), the conditional probability $P(\mathbf{d}|N_S)$ may be considered the likelihood of N_S, and is referred to as the Bayesian evidence for N_S. Since the evidence serves as a normalizing factor in Bayes' rule it can be written

$$P(\mathbf{d} \mid N_S) = \int P(\mathbf{d} \mid \mathbf{m}, N_S) P(\mathbf{m} \mid N_S) d\mathbf{m}. \qquad (7)$$

Unfortunately, numerical solution of this integral is not practical for all models sampled in the localization algorithm. Rather, an asymptotic point estimate, the BIC, is applied here:

$$-2\log_e P(\mathbf{d} \mid N_S) \approx BIC = -2\log_e L(\hat{\mathbf{m}}, N_S) + N_S \log_e N_d \quad (8)$$

where $\hat{\mathbf{m}}$ is the ML source location obtained by minimizing Eq. (6) and N_d is the number of data. As the BIC is based on the negative log likelihood, low BIC values are preferred. The first term on the right of Eq. (8) favours models with low misfits; however, this is balanced by the second term which penalizes unjustified free parameters. Minimizing the BIC provides the smallest number of acoustic sources which fits the data to within uncertainties, or, conversely, the largest number of sources resolved by the data.

The multiple-source localization algorithm developed here optimizes over the number and locations of acoustic sources, as well as complex sources strengths and noise variance at each frequency, by minimizing the BIC. This minimization is carried out by applying heat-bath (Gibbs sampling) simulated annealing with fast cooling. Source locations are treated as explicit parameters, and source strengths and variances as implicit parameters. Each iteration of the simulated annealing process consists of Gibbs sampling each location parameter as well and an attempt to either add or remove a source. Sources are added by Gibbs sampling from the conditional probability distribution defined by the existing sources, and when sources are removed the remaining sources are Gibbs sampled to compensate for the change in acoustic fields. Implementation of the implicit formulation, Eq. (6), requires a large number of complex matrix inversions which are handled efficiently using a parallel implementation of Gauss-Jordan elimination that is stable without pivoting since the matrices are diagonally dominant.

3. EXAMPLE

This section presents a (simulated) example of the multiple-source localization algorithm involving 2 submerged sources and 3 louder near-surface interfering sources, with acoustic fields recorded at $N_F = 3$ frequencies of 200, 300, and 400 Hz at a 24-hydrophone vertical array spanning a 100-m water column. The ranges, depths, and signal-to-noise ratios (SNR, taken to be constant over frequency) of the sources are as follows: source 1 (8 km, 4 m, 10 dB), source 2 (3 km, 2 m, 8 dB), source 3 (5.5 km, 2 m, 6 dB), source 4 (4 km, 30 m, 4 dB), and source 5 (6 km, 60 m, 0 dB). The source search region is 0-10 km in range and 0-100 m in depth, with from 1 to a maximum of 7 sources allowed in the search. This formulation includes a total of $2N_S(1+N_F)+N_F$ (real) unknowns (e.g., up to 51 for 6 sources), of which $2N_S$ are treated as explicit parameters and the remaining as implicit parameters.

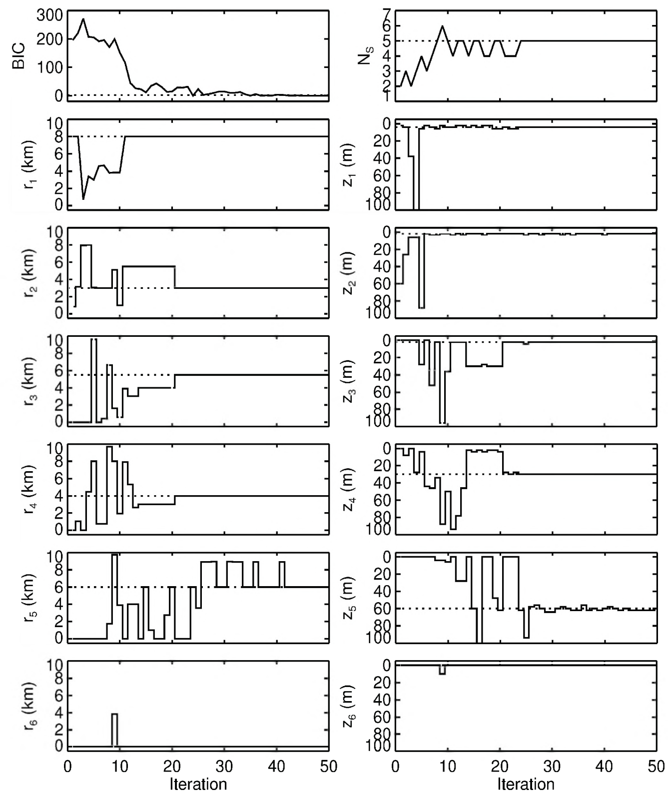

Figure 1. Inversion results as a function of simulated annealing iteration for BIC, number of sources, and ranges and depths of up to 6 sources (a maximum of 7 sources was allowed, but never accepted in the inversion). An absent source is assigned zero range and depth. The BIC is arbitrarily shifted so the minimum value corresponds to zero. Sources are ordered according to SNR. Dotted lines indicate true values.

The results of the localization are shown in Fig. 1. The BIC drops quickly (although not monotonically) and the number of sources settles into the correct value of $N_S = 5$ by about iteration 30 of the simulated annealing process. All source ranges and depths are correctly determined by about iteration 40, with the order in which the sources converge approximately following that of decreasing source SNR (i.e., the highest SNR source converges first, followed by the second highest, etc.).

REFERENCES

[1] S.E. Dosso, 2012. Acoustic localization of an unknown number of sources in an unknown environment. Canadian Acoustics, **40**, 3-12.