STRUCTURAL ANALYSIS OF MULTI-FLUID SHELL SYSTEMS SUBJECTED TO AN EXTERNAL ACOUSTIC PULSE

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1. INTRODUCTION

In our recent work [1], we have demonstrated that when an elastic shell filled with and submerged into different fluids is subjected to an external acoustic pulse, the diversity of the internal reflection and focusing phenomena is such that it leads, for certain combination of the parameters of the fluids, to a very considerable increase of the pressure in the fluid which in some cases can be as high as 110% of the peak incident pressure. It appears that such a high pressure should, at least in some instances, result in a high structural stress, an effect of obvious practical significance, and addressing such a possibility and the corresponding structural dynamics is the subject of the present study.

2. MATHEMATICAL FORMULATION AND SOLUTION METHODOLOGY

We consider a thin elastic circular cylindrical shell filled with and submerged into different fluids. We assume that the shell is thin enough, and that its deflections are small in comparison to its thickness, so that the linear shell theory can be employed; we further assume that the Love-Kirchhoff hypothesis holds true. We note that although using the Reissner-Mindlin model was shown to provide more accurate results, employing the Love-Kirchhoff model is still acceptable for the purposes of the present study.

The fluids are assumed to be irrotational, inviscid, and linearly compressible, thus the wave equations are used to model the fluid dynamics. The fluids and the shell are coupled through the dynamic boundary condition on the interface.

As was in established [2], a single most important parameter determines the appearance of the hydrodynamic fields observed in the system, namely ζ, which is defined as the ratio of the sound speed in the internal fluid to that in the external one. Changing ζ implies varying the acoustic properties of the fluids.

The problem is approached with the methodology developed in our earlier work [3-6], i.e. we apply the Laplace transform time-wise to the wave equations and then separate the spatial variables in order to obtain the pressure components as Fourier series with time-dependant coefficients. Then, the hydrodynamic and structural parts are coupled using a ID finite-difference approach.

We note that although the present approach is used, in this case, to model a structurally simple system (a single shell), it can also be successfully employed to address more complex structures [7,8]. Using a more complex model for the shell is also possible [9].

3. RESULTS AND CONCLUSIONS

A steel shell is considered with the thickness of 0.01 m and radius of 1 m, submerged into and filled with fluids of the same density but the acoustic speeds varying according to the changes of ζ. The interaction with a cylindrical pulse is analyzed, and the rate of the exponential decay is assumed to be 0.0001314 s while the initial pressure in the front is 250 kPa.

It seems suitable to first address how the high internal pressure affects the stress state of the shell. To that end, Fig. 1 shows the snapshots of the acoustic field inside the shell and the corresponding stress state of the shell at the same instant for $\zeta=0.50$. As is clearly seen from the acoustic field image, this is a scenario where the high pressure occurs in a region immediately adjacent to the shell surface, a case that is of highest practical interest.

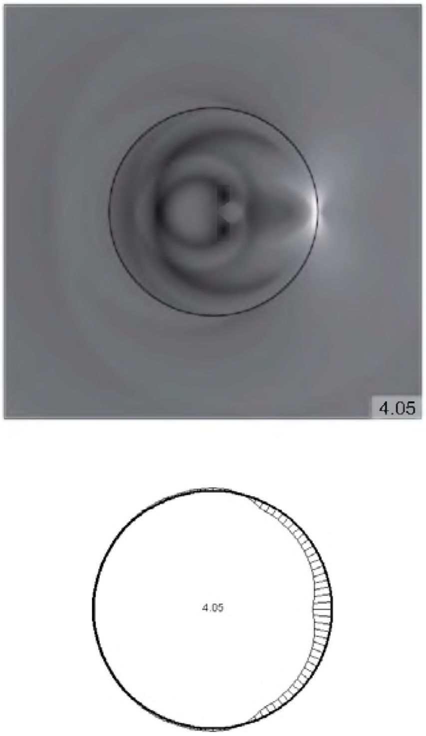

Figure 1. The internal acoustic field and the corresponding stress state of the shell for $\zeta=0.50$.

From the practitioner's point of view, however, the estimates of the maximum stress attained in the system for each combination of the parameters are a primary concern, not the detailed structure of the stress field.

In order to assess such extremities of the stress state, we analyze the highest compressive and tensile stress induced in the shell for the values of ζ that were shown to result in the most interesting internal acoustic patterns, Fig. 2 and 3. We underscore that we consider the tensile and compressive stress separately due to the fact that they have been seen to be affected very differently by the changes in the systems similar to the one at hand [7], and we will show that this is indeed a trend that is seen in the present case as well.

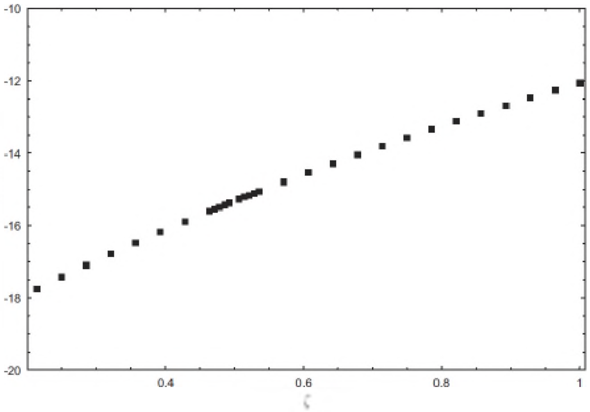

Figure 2. Variation of the maximum compressive stress depending on ζ (normalized to the peak incident pressure).

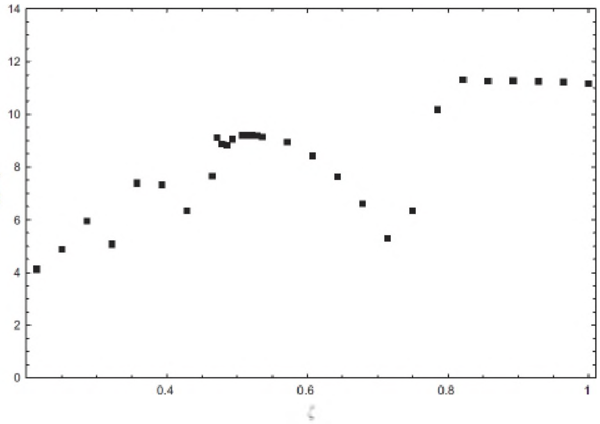

Figure 3. Variation of the maximum tensile stress depending on ζ (normalized to the peak incident pressure).

As one can see, the compressive stress exhibits a very regular pattern of change over the entire range of the values of ζ considered, and thus is not affected by the multitude of the phenomena in the internal fluid. At the same time, the tensile stress is profoundly affected by the changes of the acoustic properties of the fluids, so much so that it prompts a separate discussion of the matter.

Namely, despite the existence of the regions where the change of the peak tensile stress is rather regular, there also are regions where its changes are quite dramatic. More specifically, there are four intervals that can be identified: the intervals [0.5,0.7] and [0.82,1] which are characterized by a very regular change, the interval [0.2,0.5] where the behavior is somewhat oscillatory and no particular trend can be identified, and the interval [0.7,0.82] which is characterized by a sudden and dramatic change of the maximum tensile stress.

Within this last interval, the tensile peak is extremely sensitive to the changes of ζ: increasing ζ by only 13% (from 0.714 to 0.821) results in more than a two-fold increase of the maximum tensile stress. This sensitivity is something that the practitioner definitely needs to be aware of at the pre-design stage: even though it is very rarely a case that the properties of the fluids used in a system can be arbitrary chosen, the structural effects of varying the acoustic speeds we outlined here are too important to be ignored.

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