Accurate Modeling of the Structure of the Acoustic Field Radiated by a Submerged Cylindrical Shell Responding to an External Pulse

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1. INTRODUCTION

In recent years, a number of models have been introduced for modeling the interaction between cylindrical shells and non-stationary acoustic loads based on the Kirchhoff-Love theory of thin shells [1-5]. These models allowed for a rather comprehensive analysis of the interaction in some of the more practically interesting cases, and certain new insights into the phenomenology of the acoustic response of fluid-contacting shells followed.

At the same time, all of the models mentioned had certain shortcomings that were not essential for the general understanding of the interaction, but that presented some rather serious inconveniences when a more detailed analysis was intended. Specifically, the structure of the acoustic field re-radiated by the shell back into the fluid as it was responding to the load was found to be different than that seen in experiments [e.g. 6, 7]: the model did not reproduce the two distinctly different waves, A_0 and S_0, that have been experimentally observed.

It was therefore necessary to overcome this limitation, and there were two ways to do that. The first one was to consider the shell as a 3D solid body, thus abandoning all the simplifying assumptions of shell theories, and this has been done in [8] where a fully elasto-dynamic methodology of modeling the system in question has been developed and successfully validated.

The second way to deal with the shortcomings of the model based on the Kirchhoff-Love theory was to consider a more advanced shell theory that would still take advantage of the unique geometry of a shell, but, at the same time, would be more accurate in terms of reproducing the structural behavior of the shell (and, therefore, the dynamics of the radiated field as well), perhaps at the expense of its computational attractiveness as it compares to its Kirchhoff-Love counterpart. Such an undertaking is the subject of the present study.

2. MATHEMATICAL FORMULATION AND SOLUTION METHODOLOGY

We consider a thin evacuated elastic circular cylindrical shell submerged into fluid, and model it using the Reissner-Mindlin theory of shells [9]. The fluid is assumed to be irrotational, inviscid, and linearly compressible, thus the wave equations are used to model the fluid dynamics. The fluids and the shell are coupled through the dynamic boundary condition on the interface.

The problem is approached with the methodology developed in our earlier work [1-5], i.e. we apply the Laplace transform time-wise to the wave equations and then separate the spatial variables in order to arrive at the expressions for the transforms of the internal and external pressure in a form of a series of modified Bessel functions of the first (internal fluid) and second (external fluid) kind. The pressure is then obtained as a Fourier series with time-dependant coefficients which, for the radiation pressure, depend on the unknown normal displacements of the shell.

Then, the same series form is used for the shell displacements and, substituting them into the shell equations, we arrive at the systems of the ordinary differential equations for each of the displacement harmonics. The systems are then approached numerically, using a finite-difference scheme, and the resulting normal displacement is used to compute the radiation pressure.

We note that although the present approach is used here to model a single shell, it can also be successfully employed to address systems consisting of several structures, for example a shell with a rigid core [10,11].

3. RESULTS AND CONCLUSIONS

A steel shell is considered with the thickness of 0.03 m and the radius of 1 m, submerged into water. The interaction with a cylindrical external acoustic pulse is analyzed, and the rate of the exponential decay is assumed to be 0.0001314 s while the initial pressure in the front is 250 kPa.

Fig. 1 shows three images of the radiated field for the chosen shell and pulse using three different models. The first image is that produced by the experiments [7], and it clearly shows the existence of two radiated waves, the labeled S_0 wave, or symmetric Lamb wave, and the A_0 wave, or pseudo-Rayleigh wave. The second image is numerically simulated and it is produced by a model based on the Kirchhoff-Love theory (KL model). The third image is numerically simulated using the Reissner-Mindlin shell theory (RM model).

As is apparent from the images, the KL model, although correctly reproducing the overall shape of the radiated field, fails to accurately reproduce its details, the drawback we mentioned earlier. The RM model, however, correctly reproduces both the S_0 and A_0 waves, thus eliminating the issues inherent to the KL model. At the same time, still being a shell model, it relies on certain simplifying assumptions regarding the structural behavior of the shell, and thus is very attractive from the computational point of view. More specifically, simulations of the stress state of the shell based on this model take only 13% longer than those based on the KL model, the latter has been demonstrated to be highly computationally efficient in a number of contexts.

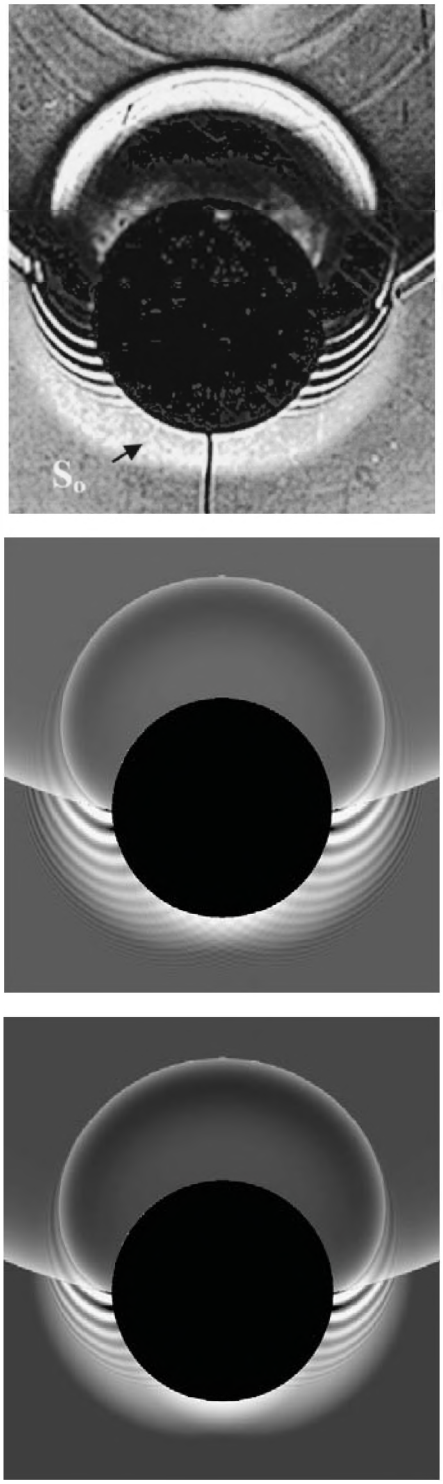

Figure 1. The radiated field for a steel shell with the thickness-to-radius ratio of 0.03 submerged into water: experiments [7], top image; KL model, middle image; RM model, bottom image.

Along with the qualitative validity of the simulated results, their quantitative validity had to be established as well. To that end, we compared the numerically simulated time-histories of the acoustic pressure at several points of the fluid to the available experimental time-histories for similar systems, and observed a very good match for the S_0 wave. At the same time, due to the lack of the available experimental data, we have not so far been able to assess the quantitative accuracy of the modeling of the A_0 wave.

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ACKNOWLEDGEMENTS

S.I., K.W., R.M. and M.M. acknowledge the financial support of the Natural Sciences and Engineering Research Council (NSERC) of Canada. H.S. acknowledges the financial support of the Fundação para a Ciência e a Tecnologia (FCT) of Portugal. S.I. also acknowledges the financial support of the Killam Trusts and Dalhousie University.