SIMULATION STUDY OF JOINT TRANS-DIMENSTIONAL BAYESIAN INVERSION OF SCATTERING AND REFLECTION DATA

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1. INTRODUCTION

Ocean acoustic reverberation modelling and sonar performance prediction in shallow waters require good estimates of seabed geoacoustic parameters and scattering parameters defining seafloor roughness. Direct measurements of these parameters are time consuming and expensive, and it may be advantageous to estimate *in-situ* seabed parameters based on indirect measurements, e.g., as the solution to an inverse problem. This paper develops a joint trans-dimensional (trans-D) Bayesian inversion approach which is applied to synthetic seabed scattering and reflection data with the goal of determining the ability of such data to resolve geoacoustic and scattering parameters.

2. METHOD

Bayesian inversion requires specifying the posterior probability density (PPD, product of the prior distribution and likelihood function), and a method to sample PPD. The remainder of this section gives an overview of the creation of the synthetic data and of the Bayesian inversion approach as it is applied here; more general and complete descriptions of Bayesian inversion are given elsewhere[1,2].

2.1 Synthetic Data

The scattering data represent monotonic back-scatter strengths generated from Jackson's perturbation-theory scattering model[3] assuming a two-dimensional seabed roughness power spectrum (the von Karman spectrum) [4]

$$W(\mathbf{K}) = \frac{w_2}{\left(|\mathbf{K}|^2 + K_0^2\right)^{\gamma/2}}, \tag{1}$$

where w_2 is the spectral strength, K_0 is the spectral cut off, and γ is the spectral exponent. The vector $\mathbf{K}$ is the transverse component of the incident wave vector (i.e. $|\mathbf{K}|=k_0\cos(\theta)$, where θ is the incident grazing angle and k_0 is the wave number in the ocean). Data were created over an angular range of 6–24° at six frequencies (600 Hz, 900 Hz, 1200 Hz, 1800 Hz, 2400 Hz, and 3600 Hz). Gaussian-disturbed errors are added to the data which are correlated over angle but independent between frequencies.

The reflection data correspond to spherical reflection coefficients calculated recurvsively over a layered seabed[5]. Reflection data were generated over an angular range of 20–85° and averaged over 1/3-octave bands for six centre frequencies (630 Hz, 800 Hz, 1000 Hz, 1500 Hz, 2500 Hz, and 3600 Hz). The angular spacing between data points is non-uniform and increases with angle from approximately 0.3–10°, which is typical of experiment measurement techniques[6]. Data errors are positively correlated (over angle) with correlation between to data points decreasing exponentially with angular distance.

2.2 Posterior Probability Density

The PPD contains all information considered in Bayesian inversion, and can be expressed using Bayes rule as

$$\mathrm{PPD}(\mathbf{m}_j \mid \mathbf{d}) = \frac{\mathcal{L}(\mathbf{d} \mid \mathbf{m}_j)\pi(\mathbf{m}_j)}{\mathcal{Z}}, \tag{2}$$

where $\mathcal{L}$ is the likelihood function, π is the prior distribution, and $\mathcal{Z}$ the evidence. The vectors $\mathbf{d}$ and $\mathbf{m}_j$ represent the data and the model parameters, where subscript j indicates the model dimension which is variable in trans-D inversion. Here the number of sediment layers is treated as unknown and sampled in the inversion.

The prior distribution represents information known about the model before the introduction of the data. The prior distribution used here is

$$\pi(\mathbf{m}_j) \propto \frac{(j-1)!}{z_B^{(j-1)}} \frac{H^j}{(\Delta c \Delta\rho \Delta\alpha)^j}, \tag{3}$$

where Δc, $\Delta\rho$, and $\Delta\alpha$ are the range of possible sound velocity, density, and attenuation values for a given sediment layer; H is a normalizing constant that accounts for the assumed correlation between geoacoustic parameters; z_B is depth of the basement. The prior distribution can be formulated proportionally since Bayesian inversion considers only the ratio of prior distributions. The remaining parameters, such as w_2, K_0, and γ have uniform priors and are absorbed by the proportionality.

The likelihood function describes both the physics (forward model) and the data error statistics assumed in the inversion. As this work describes a simulation study, the same forward model and data error statistics used to create simulated data are assumed in the inversion.

2.3 Sampling Scheme

It is not in general possible to interpret the PPD in an analytic manner. Thus in Bayesian inversion it is common to sample the PPD using Markov-chain Monte Carlo algorithms; here, the reversible-jump Markov-chain Monte Carlo (rjMCMC) algorithim[7] with parallel tempering[8] is used. The sampled PPD is then interpreted in terms of its

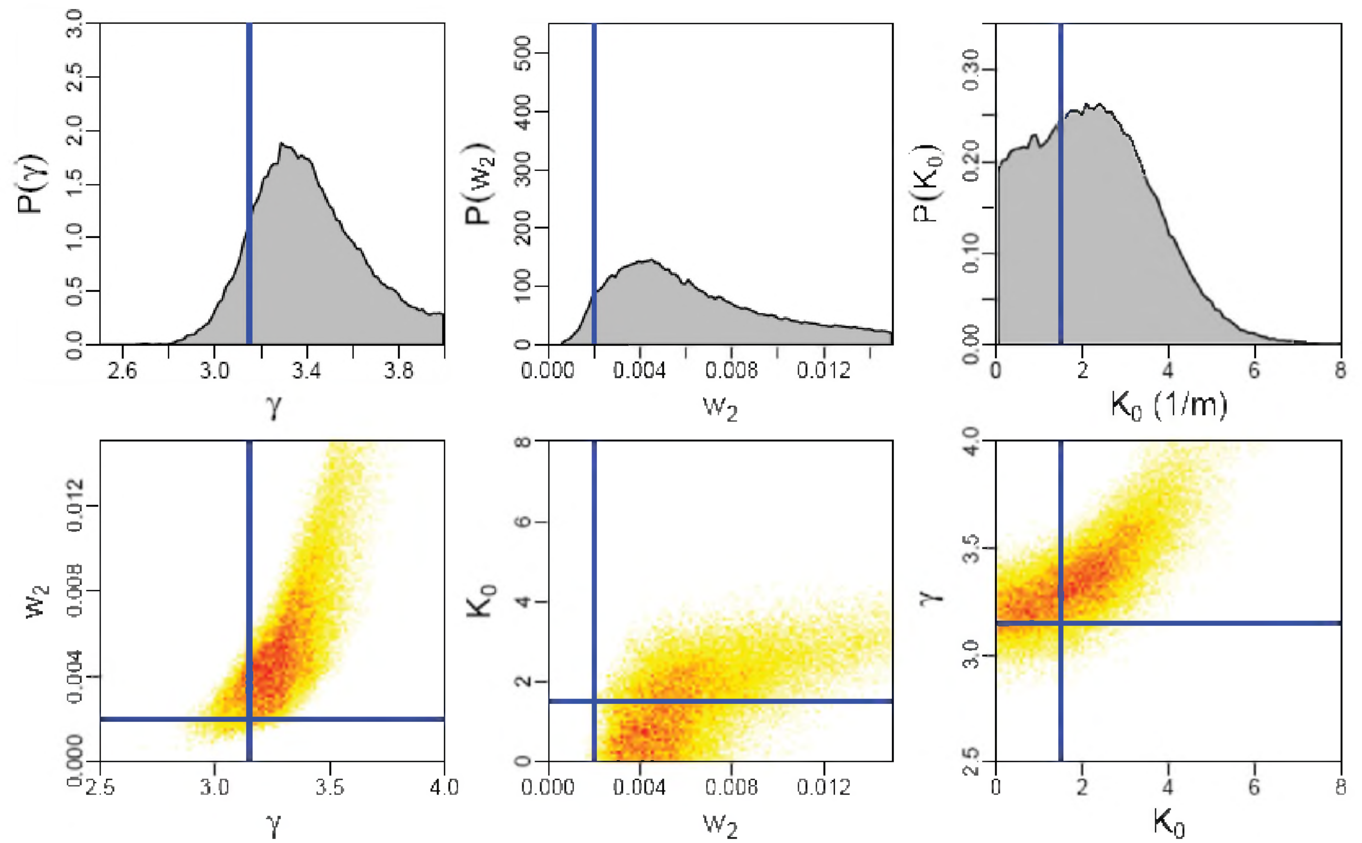

FIG. 1. Top: one-dimensional marginal posterior distributions of the scattering parameters. Bottom: two-dimensional marginal posterior distributions of the scattering parameters. The horizontal and vertical lines indicate the true value.

moments, parameter uncertainties (variances, marginal distributions, credibility intervals), and parameter inter-relationships (correlations and joint marginal distributions).

To adequately approximate the PPD 600,000 models were sampled from it using the rjMCMC algorithm. These are thinned by one third to reduce sample correlation; only these remaining samples are considered here.

3. RESULTS

The one- and two-dimensional marginal distributions of the scattering parameters (w_2, K_0, and γ) for the inversion are shown in Fig. 1. The parameter distributions are centred near the true values, and the uncertainties indicate a useful level of resolution of the roughness spectrum. Geoacoustic parameters are also well resolved, but are not shown here due to space constraints. This work indicates that Bayesian inversion is an adequate way of determining the acoustic scattering properties of marine sediment.

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