DEPARTURES FROM THE ACOUSTICAL PARAMETERS IN THE INTONATION OF SOUTH INDIAN MUSICAL INTERVALS

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ABSTRACT

All evolved music cultures have engaged in the study of musical intervals. Indian musical intervals are melodic. Indian music was not structured acoustically[1]. But in recent times scholars began to give precise acoustical values for the musical notes. This paper shows the departures from such acoustical parameters and explores the pitch profiles characteristic to the South Indian *rāga* music. It demonstrates the dynamic variability in the intonation of the South Indian musical intervals. A holistic view incorporating hearing and perception are vital to the understanding of culturally sensitive idiomatic pitching of the musical intervals.

SOMMAIRE

Toutes les cultures avancées se sont intéressées de tout temps à l'étude des accords et intervalles musicaux. Les accords de la musique indienne sont mélodiques. La musique indienne n'était pas structurée d'un point de vue acoustique. Mais avec le temps les chercheurs ont commencé à attribuer des valeurs acoustiques précises aux différentes notes musicales. Cet article montre le point de départ de tels paramètres acoustiques et s'intéresse aux caractéristiques du profil mélodique de la musique râga de l'Inde du Sud. La variabilité changeante de l'intonation des intervalles musicaux de l'Inde du Sud sera mise en évidence. Une perception holistique, incorporant l'ouïe et la perception, est vitale pour la bonne compréhension des accords musicaux dans leur contexte idiomatique et culturel.

Editor's NOTE: The superscripts refer to explanatory notes provided in Appendix A.

1. INTRODUCTION

This paper examines the pitching of musical intervals in the classical music tradition of South India. The extant theory of intonation demands fixed frequency values for the musical intervals. But based on the evidence provided by the computer pitch analysis of select audio samples, departures from such acoustical parameters can be clearly demonstrated. In section 2, a brief background on the ancient system of Indian music, systemic changes in later times and the adaptations of the western concepts of intonation are provided to set the stage for the current discussion. Section 3 talks about the melodic nature of the South Indian musical intervals and explains why certain basic harmonic intervals are used as well. This section briefly alludes to hearing and perception, and discusses aspects of consonance and neurological recognition of pitch contours. Section 4 presents the empirical pitch analysis. It provides select pitch plots along with the explanations of these pitch profiles accompanied by a table capturing the departures from the theoretically expected fixed frequency values given by the modern scholars. Section 5 elaborates on the departures from the acoustical parameters. This section talks about the *intonational imperatives*, elucidating the dynamic variability in intonation in the performance of South Indian *rāga* music. This section also examines the influence of the accompanying *tambūra* [2] on the intonation.

Section 6 summarizes the conclusions. The appendices with the explanatory notes give further information.

The word 'Indian' specifically refers to 'South Indian' music because the supporting evidence provided in this paper is the empirical pitch analysis of South Indian *rāga*-s. Also, the word *rāga* specifically means South Indian or *Karṇāṭaka* (Carnatic) *rāga*.

2. BACKGROUND

Musical intervals in Indian music have been traditionally expressed in terms of the number of *śruti*-s between them. In the ancient Indian music, *śruti*-s were a means of measurement of *svara*-s (Indian musical notes). In the modern parlance we might refer to them as microtonal octave divisions. Ancient musical treatises in Sanskrit explained the musical intervals in the context of the theory of 22 *śruti*-s [3]. (Refer also to Appendix B). Bharata, in his *opus* the *Nāṭyaśāstra*, has explained the theory of 22 *śruti*-s. Subsequent treatises reiterated this theory but with varying interpretations. The modern scholars, in the last 150 years or so, have given a mathematical interpretation to the theory of 22 *śruti*-s and wrongly attributed them to Bharata for authenticity.

In the ancient fixed-interval *grāma* system, *śruti* was a means to express the intervals as well as the *svara*-s. In the *Sa-grāma*, *Sa*, *Ma* and *Pa* were referred to as *catuḥśrutika* (four-*śruti*-ed) *svara*-s, *Ri* and *Dha* as *triśrutika* (three-*śruti*-ed) *svara*-s and *Ga* & *Ni* as *dviśrutika* (two-*śruti*-ed) *svara*-s. These intervals were fixed. Different scales were generated by shifting the tonic. In the current fixed-tonic *mēla* system, the intervals are variable. According to Powers (1970: P62), the original intent and usage of the word *śruti* had lost its currency as the system changed from *grāma* to *mēla*. But modern scholars[4] continued to use *śruti* in the same old sense. The ancients never derived *śruti*-s mathematically nor were they concerned about the acoustics. This is evident from a chronological study of the original ancient texts in Sanskrit. The seven notes in ancient music were positioned within the 22 *śruti*-s' schematic. *Śruti* was only a conceptual measure; not an acoustical quantity and it was never expressed in the form of m/n until the modern times.

Pythagoras based his tuning system on the octave (2:1) and used the simple relationship of the pure fifth (3:2) to arrive at the other intervals expressed in ratios in the form of m/n (Barbour, 1953, *xi*). The treatment was mathematical. Early Indians like Bharata positioned the musical notes (*svara*-s) in terms of the *śruti* intervals. The treatment here was not mathematical. As Rowell (1992, P52) points out, ancient Indian musicians described the musical sound by the 'word' whereas the Greeks and the medieval Europeans used the 'number' to describe it. The ancient Indians considered the musical sound as a continuous and dynamic flow of the vital breath. The Greeks on the other hand considered it as a static material substance. One understood it as a quality, while the other took it as a quantity.

In the modern period (i.e. post seventeenth century) however, alien western concepts of acoustics and Pythagorean and Just tuning systems have been forcibly applied to the study of Indian *śruti* intervals assigning precise mathematical values. These adaptations have caused irreconcilable discrepancies. They began to treat *śruti*-s as physical quantities and assigned fixed values. (Refer also to the table of commonly accepted modern *śruti* values in Appendix B). The horizontal lines in the pitch plots, presented in Figures 1 through 14, indicate the theoretically expected frequency values from the table of Appendix B. Despite such adaptations by the modern scholars, the original intent of the 22 *śruti*-s is not compatible with any of these western tuning concepts. There is no consistent predilection for Pythagorean or Just or any other 'natural' intervals (Komaragiri, 2005, P92). This is the case even in the western music (Burns, 1999, P246). Today however, notes are positioned on the musical instruments, particularly the fretted stringed instruments, in line with the natural consonance, but determined aurally. The study of intonation based on the concept and terminology of *śruti* remained academic and confined to musicologists. Latter-day musicologists like Veṅkaṭamakhī[5] conceived music in

terms of 12 notes (with 16 *svara* names)[6], excerpted from the 22 *śruti*-s. The '12 notes' system is currently in practice. North Indian musicians always talked about their music in terms of 12 notes.

Even if the musical intervals are studied in terms of mathematical ratios, mathematical simplicity in itself is almost irrelevant to the study of acoustics or of music (Benade, 1976, P264), because of the complexity of *rāga* music. In other words, if the pitch profiles were to be mathematically modeled, simple ratios in the form of m/n will not suffice. The non-linear, context-dependent dynamic variability of musical pitches will necessitate higher order mathematical formulations. The individual artistic idiosyncrasies (human element) will further complicate the modeling.

Musical intervals are essentially of two types; harmonic and melodic. Harmonic intervals are formed between two simultaneous notes whereas melodic intervals are formed between two sequential notes (Lloyd, 1963, P125). Melodic intervals constitute a distance, whereas harmonic intervals can be expressed in ratios in the form of m/n for non-zero positive integers (Levarie, 1980, P17). Because melodic intervals do not constitute beating upper partials, mathematical rigidities do not apply to melodic intervals even if the notes are formed from the natural harmonic series (Benade, 1976, P293). This has been amply corroborated psycho-physiologically. For instance, beats do not occur on the basilar membrane (inside the inner ear) between successive notes (Moore, 2003, P227). Musical intervals cannot be defined entirely with harmonic ratios of their frequencies and with physical concordance (zero / minimum beats) of the upper partials. The wholesome process of pitch perception is essential to the understanding of musical intervals. It should be noted that the physical concordance is not the same as the musical consonance (Lloyd, 1963, P136).

3. SOUTH INDIAN MUSICAL INTERVALS

South Indian musical intervals are melodic, as are the intervals of North Indian music. Although mathematical rigidities do not apply to melodic intervals, certain basic intervallic relationships, such as the octave, the fifth, the fourth and the major third hold true for several reasons. The *tambūra* (*tānpūra*), the fret-less stringed instrument that provides the tonic key also provides certain basic natural harmonics such as the octave, the fifth, major third etc. Training with the *tambūra* accompaniment tunes up and conditions the tonotopic[7] organization. The tonotopic pitch mapping in the primary auditory cortex actually alters and firms up upon deliberate learning and training. This tonotopic organization is retained in memory (Weinberger, 1999, P55). And the memory guides the perception of melodic intervals (Burns, 1999, P230), particularly if the delay between the two successive tones is less than about ½ second (Benade, 1976, P288).

Other than these basic harmonic intervals, South Indian musical intervals are basically melodic and are therefore flexible. Since the tonotopic organization is based on pitch and not on frequency (Weinberger, 1999, P62), the intervallic relationships follow individual learning patterns and cultural contexts as opposed to frequencies of the acoustical stimuli. The neuronal responses are driven by the type of stimuli (Pickles, 1988, Chap 7; Moore, 2003, P50) and therefore psycho-physiologically, melodic intervals are pitched and perceived differently from the harmonic intervals and are not solely dependent on the physics of vibrations.

The resonance region on the basilar membrane shifts by the same distance for a given pitch transition regardless of the octave (i.e. frequency range) (Roederer, 1995, P26). That is, we hear logarithmically. The ear's non-linearity has been established physically, physiologically, anatomically (masking and facilitation), psychologically and psycho-acoustically (Komaragiri, 2005, P106). This non-linearity in hearing renders simplistic mathematical ratios, which pertain to pure sinusoids, untenable. Musical intervals have complex waveforms comprising inharmonic upper partials. It may be interesting to point out that combination tones, also sometimes referred to as heterodyne frequencies are created due to the non-linearity in the cochlea (Pickles, 1988, P154). These heterodyne beat frequencies are helpful in the tuning of musical instruments. But they relate to the harmonic partials. As explained above, the composition of the melodic intervals is different from that of the harmonic intervals; the zero-beat condition does not pertain to the melodic intervals.

Musical scale and consonant harmonic intervals are not compatible in an exact sense (Pierce, 2001, P177). That is, the progression of consonant intervals will not lead to the perfect octave. Scale on the other hand is a collection of tempered notes arranged within the octave. Such tempered notes are approximations of the consonant intervals for practical purposes. This is not an issue for equal temperament. But Indian intervals being melodic and therefore being flexible and contextual can be reconciled artistically with the scale of the *rāga* set within the octave. It should however be emphasized that the Indian *rāga*-s are *not* mere scale patterns. They are characterized by melodic intervals and consequently by melodic intonation. In other words, *rāga*-s are characterized by definite pitch profiles. They are not arbitrary. A set of *rāga*-specific reference pitches and pitch profiles can be empirically obtained for specific conditions. Indian melodic intervals transcend rigid pitch fixations in performance.

The focus of this paper is on the departures from fixed frequency values in intonation and not on consonance. But, it may be germane to qualify musical consonance, as intervals of a *rāga* must have musical consonance. Musical consonance has several layers. In a fixed-tonic scale system, the tonic provides the overall reference. All the other notes therefore become intervals with reference to the fixed tonic, provided by the accompanying *tambūra*. The pitch plots

presented in Figures 1 through 14 pertain to these intervals. Musical intervals are also perceived between any two notes in mutual relationship. In the rendering of a *rāga*, the melodic intervals between any two notes are pitched with the first of the two notes as the reference. This is one layer of musical consonance. Another layer of musical consonance is that of the intervals within a characteristic phrase. Pitch plots of Figures 13 and 14 give a preliminary glimpse of the intervallic balance. Rao (2000, P81-83) also supported this view for the North Indian music. As already noted above, this does not submit to the frequency fixations of the whole-number ratios. Yet another layer of consonance is the mutual setting of the different phrases within the overall balance of the *rāga*. That is, an inherent balance in the musical intervals, judged aurally, which varies dynamically within the overall *rāga* gestalt. These multiple layers of musical consonance are not hierarchical.

The characteristic phrases of *rāga*-s have recognizable pitch contours. It is pertinent to note that the functional connections among neural cells are not fixed but rather depend on the pattern of tone sequences comprising a contour (Weinberger, 1999, P75). Neuronal groups are responsive to the contour despite the variability in musical intervals within a given contour or a phrase. The responsiveness of the same tone differs from contour to contour, at least in some cases (Weinberger, 1999, P73), indicating that the musical intervals vary contextually rather than merely acoustically. This implies that a particular phrase in a *rāga* or an overall sequence of musical intervals comprising a contour is recognized even if the musical intervals comprising the phrase vary, as long as the overall balance is maintained to preserve the phrase or the contour.

4.   EMPIRICAL PITCH ANALYSIS

The pitch plots presented in Figures 1 through 14 have been excerpted from Komaragiri (2005). These pitch plots pertain to the South Indian music. A similar analysis may be extended to the North Indian classical music also. The pitch plots of the figures show the pitch profiles of various *rāga*-s are marked with the following abbreviations: MBK for M Balamurali Krishna (vocal); TNK for T N Krishnan (violin) and NRK for N Ravi Kiran (citra-vina). *Gaula, Sāvēri, Hindōlaṃ* (or, *Mālkauns*) and *Haṃsānandī* (or, *Sōhini*) are the names of the *rāga*-s presented in this paper. The seven notes in the Indian music are represented by *Sa* (the tonic), *Ri* (the second), *Ga* (the third), *Ma* (the fourth), *Pa* (the fifth), *Dha* (the sixth) and *Ni* (the seventh). The actual names of the musical notes used in the South Indian music are given in the Appendix B. Most modern musicologists have acoustically interpreted the ancient theory of 22-*śruti*-s pertaining to the *grāma* system, deriving 22 harmonic frequency values, measured from *Sa*. These theoretical frequency values are also given in Appendix B.

The pitch plots, presented in Figures 1 through 14, are drawn with pitch in Hertz along the ordinate and time in

seconds along the abscissa. The horizontal lines are the expected theoretical values (the acoustical parameters). They are indicated both in frequency ratios and in cents [8]. The measured pitch values are also given in cents, c, for easy comparison. See Appendix B for all the cent values. Table 1 given below summarizes the empirical pitch analysis results. The variability in the mean pitch values and their departures from the acoustical parameters can be clearly seen. The negative sign indicates dipping below the datum *Sa*. The datum *Sa* for MBK was 123 Hz; for TNK, 350 Hz and for NRK, 199 Hz. The variation is datum *Sa* was up to 20 cents. There is variation even in the octave, sometimes measuring less than 1200 cents. The intensities measured were 60 dB, 45 dB and 40 dB for MBK, TNK and NRK respectively.

Table 1: Expected Acoustical Values and Departures

PP No	Mean	Max	Min	Range
Ri 256/243 = 90 cents & 16/15 = 112 cents				
01	-12	146	-74	220
02	23	121	-9	130
03	24	74	-5	79
04	88	129	9	120
05	23	77	-19	96
06	67	126	6	120
Dha 128/81 = 792 cents & 8/5 = 814 cents				
07	724	798	687	111
08	796	856	745	111
Ni 16/9 = 996 cents & 9/5 = 1018 cents				
09	1005	1025	977	48
10	1056	1122	1002	120
Ni 15/8 = 1088 cents & 243/128 = 1110 cents				
11	1078	1129	1032	97
12	1162	1214	1111	103

Several combinations of the same artiste rendering the same intervals and different artistes rendering the same intervals are selectively presented here. Figures 1 through 4 show the departure from the acoustical values for the interval *Ri* in the *rāga Gaula*. Intervals are measured from the tonic, *Sa*. The variation in *Ri* is about a semitone. There is variation even within the same musical context (*Ma-Ri-Pa-Ma*). Figures 5 through 8 show similar departures in the intervals *Ri* and *Dha* in the *rāga Sāvēri*. The variation in the two *Dha*-s is about 70 cents but NRK's *Dha* ranged from 724 cents to 817 cents. Figures 9 through 12 show the two varieties of *Ni*, the seventh note. Figure 10 showed a departure of 38 cents above the reference and overshooting

the higher *Ni*. This raised pitch is in spite of the downward movement of *Ni* in the musical context *Sa-Ga-Ma-Ni-Dha*. Perceptually, *Ni* is well pitched within the pentatonic *Hindōlaṃ* (or, *Mālkauns*), despite having a quarter tone difference between the two artistes. Figure12 showed a departure of 52 cents above the reference. And this is just 38 cents below the octave. The mean value of the note itself ranged over 54 cents. But such raised pitch did not occur in MBK's rendering in Figure 11. In addition to the variations in the measured mean pitch values for various intervals by various artistes, even for each individual measurement there is a significant range oftentimes covering a semitone (See Table 1). This shows that the characteristic pitch movements (contours) are essential to the Indian *rāga* music instead of the individual pitch values.
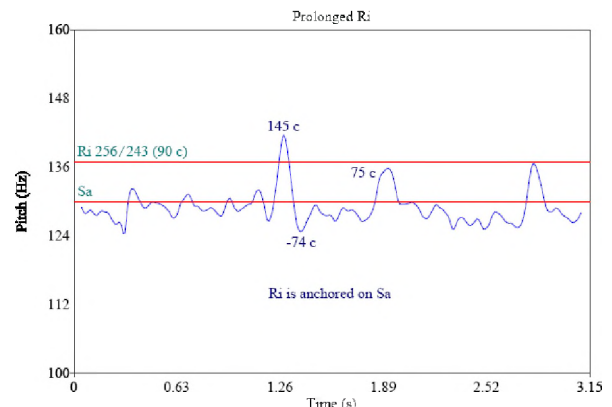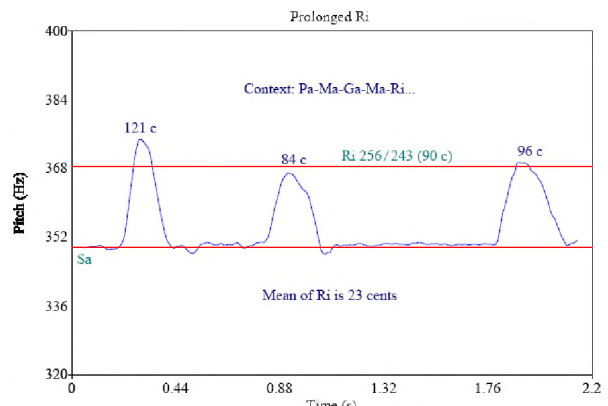

Figure 1. MBK-*Gaula-Ri*
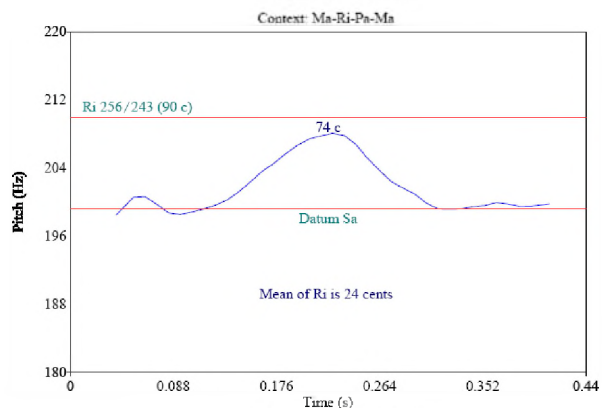

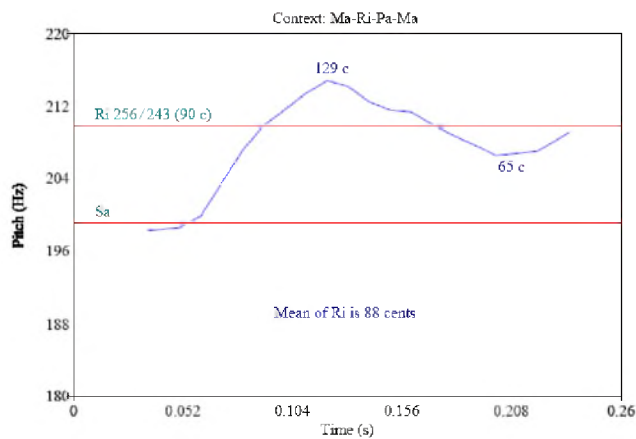Figure 2. TNK-*Gaula-Ri*

Figure 3. NRK-*Gaula-Ri* - 1

Figure 4. NRK-*Gaula-Ri* - 2
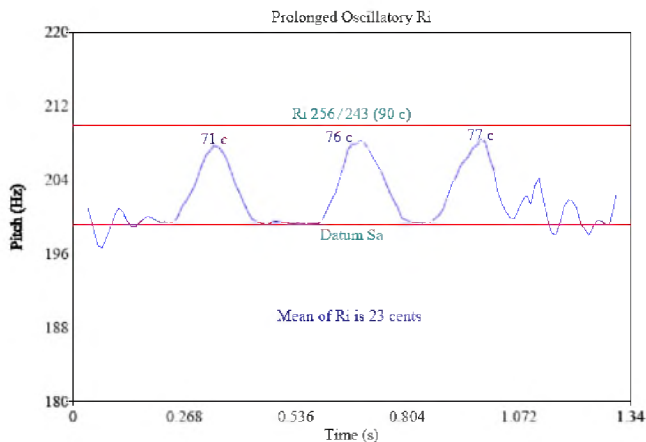
Figure 5. NRK- *Sāvēri--Ri* – 1

Figure 6. NRK- *Sāvēri--Ri* - 2

Ma-Ri with a prolonged *Ga* and Figure 14 shows the phrase *Ri-Ga-Ma-Ri* with a shorter *Ga*. And, the difference of 33 cents in the *Ri-Ma* interval between these two phrases, also illustrates the variability in the intonation of these intervals. The datum *Sa* for this particular rendition measured at 130 Hz.
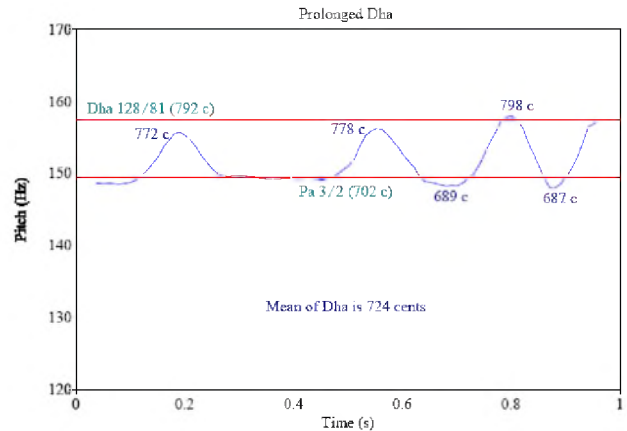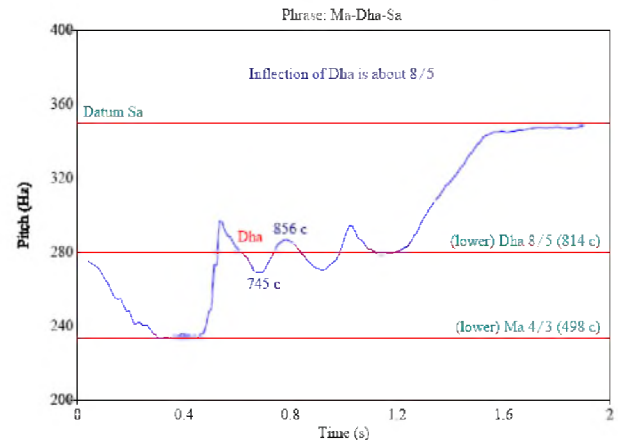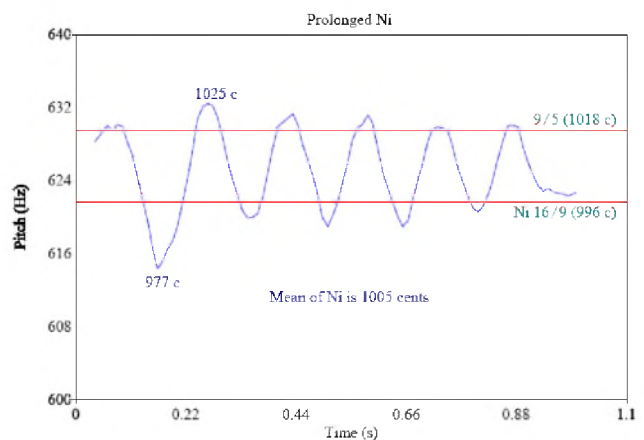

Figure 7. NRK- *Sāvēri--Dha*

Figure 8. TNK- *Sāvēri--Dha*

Figure 9. TNK- *Hindōlam-Ni*

In addition to the study of intervals, it is interesting to notice the intervallic balance in similar phrases within the same rendition. For example, Figures 13 and 14 show similar phrases in the same *rāga Gaula,* sung by the same artiste, MBK, in the same rendition. These two plots illustrate the intervallic balance. In Figure 13, *Ri-Ga* measured a mean of 369 cents and *Ga-Ma*, 73 cents. In Figure 14, *Ri-Ga* measured a mean of 333 cents and *Ga-Ma*, 142 cents. That is, in Figure 13, as *Ri-Ga* interval is higher, *Ga-Ma* interval is rendered lower than the corresponding interval in Figure 14, showing a propensity for intervallic balance within similar phrases. Figure 13 depicts the phrase *Ni-Sa-Ri-Ga-*

This type of intervallic balance can be seen in several other instances. But when such balance is not seen in the immediate context, larger contexts might have to be examined, which is outside the scope of the current paper. It may also be noted here that there is similarity in the overall

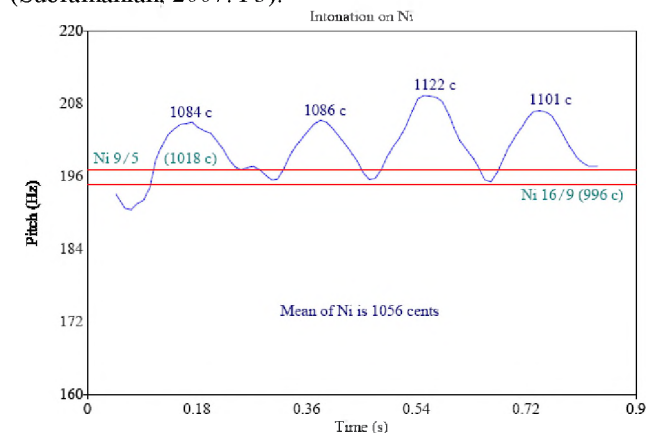pitch movements of similar phrases when depicted visually (Subramanian, 2007: P3).
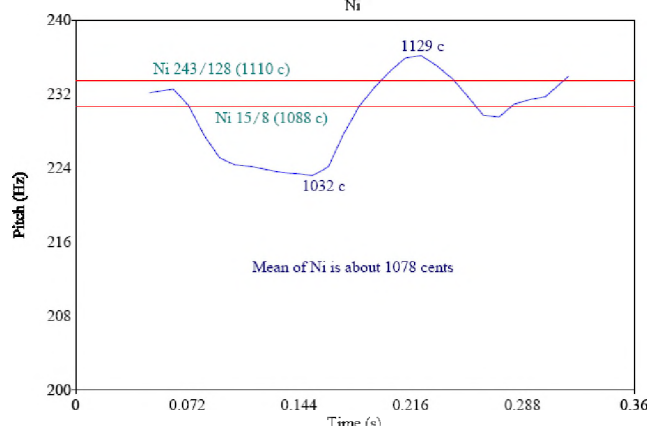

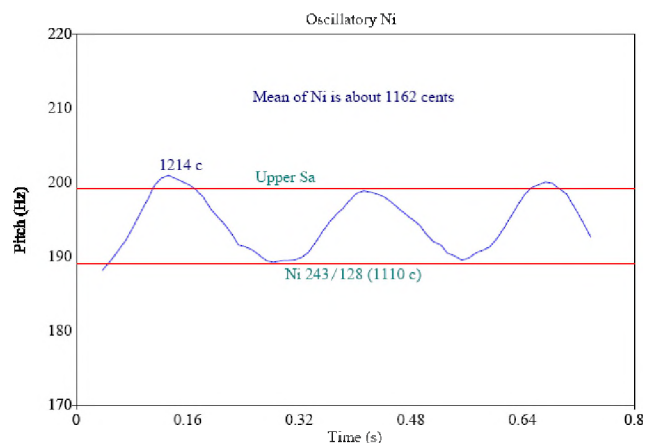Figure 10. MBK- *Hindōlaṃ-Ni*

Figure 11. MBK- *Haṃsānandī–Ni*

Figure 12. NRK- *Haṃsānandī–Ni*

5. DEPARTURES FROM THE ACOUSTICAL PARAMETERS

Departures from the theoretically calculated acoustical values are clearly demonstrated in the South Indian *rāga* music. The intervals vary dynamically depending upon the artistic individuality and the musical context. The notes are intoned differently in different *rāga*-s and differently in different phrases within the same *rāga*. Different artistes intone the same notes differently in a given *rāga* and

differently at different times when repeated. All these different cases constitute different musical contexts. Intonation in the South Indian *rāga* music is not acoustically based; it does not submit to the mathematical rigidities.
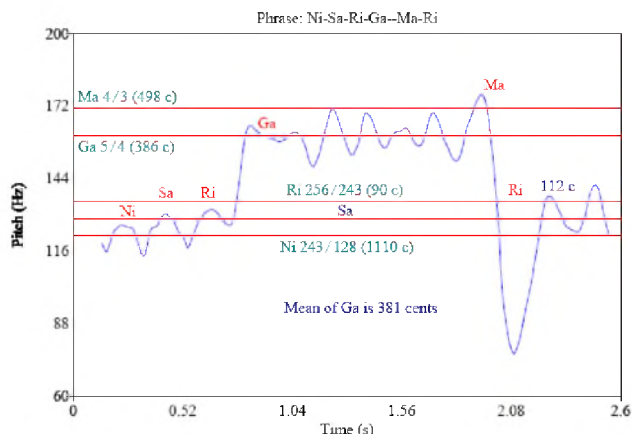

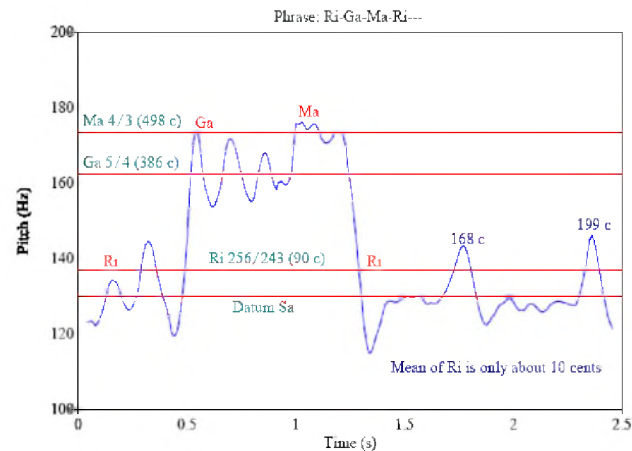Figure 13. MBK- *Gaula*-Phrase-1

Figure 14. MBK- *Gaula*-Phrase-2

The inverse relationship between periodicity (time) and frequency (pitch) places an unavoidable restriction on the precision of pitch measurement and perception and therefore on the intonation. Physiologically also there is inverse relation between temporal and spatial resolutions (Winckel, 1967, P5 & P121). Further, the sensitivity areas of say, the two types of thirds overlap and as a result, depending upon the context, either of the two types of the third is perceived (Winckel, 1967, P124-125). That is, in the absence of one whole tone, it would be very difficult to know precisely which of the two whole tones is being perceived (Levarie, 1980, P203). Therefore the distinction between any two types of the whole tones, 9/8 and 10/9, for example, is unrealistic because of the uncertainty in the perception of their intervallic relationships in an absolute sense. Such uncertainty defies mathematical precision. The pitch plots presented above illustrate this succinctly.

The process of determining a melodic interval is not a basic sensory process, but it involves memory and learning within a cultural context. There is considerable latitude in the acceptance of melodic intervals as they carry the appropriate

melodic information within this latitude (Burns, 1999, P231). In Indian music, the accompaniment of *tambūra* (*tānpūra*) restricts this latitude for certain basic harmonic intervals such as the octave and the fifth.

The accompaniment of the *tambūra* guides intonation as indicated above. The *tambūra* might have been introduced around the fifteenth century (Deva, 1981, P61). The modern-day *tambūra* has a curved bridge and the strings pass over a thread, called the *jīvam* or *juari*. As a result, the strings actually leave the bridge and make contact again repeatedly (Modak, 1977, P8). Thus the string length continually changes, resulting in a continual change in the fundamental frequency. Variations in the fundamental are between 0.5 to 1% (Modak, 1977, P14). Addition of this thread decreases the strength of the fundamental in relation to the harmonics, as also pointed out by C.V. Raman and Modak. The fundamental is not always dominant and at times can even be missing (Deva, 1981, P32). Deva says that the 3rd and the 5th harmonics are the strongest. Spurts of certain partials (harmonic and inharmonic) can be heard as well (Deva, 1981, P35). When the fundamental is missing, the ear will provide it because of the non-linear nature in its perception (Roederer, 1995, P45). Modak had shown that the fundamental and the harmonics build and decay at different rates and that the phase difference between them continually changes. As a result of phase fluctuations, even the second harmonic, which is the octave, is not exactly in the ratio of 2:1, and is out of phase with the fundamental. This is attributed to the curved shape of the bridge. Raman CV (1921, P471) had shown that the strings of the *tambūra* do not obey Young and Helmholtz law [9]. The *tambūra* seems to produce an envelope of continually changing inharmonic partials. The *tambūra* however demands a final resolution to the tonic.

The inharmonicity in the accompanying *tambūra* is one of the reasons why precise harmonic frequency ratios lose meaning. Tempering of the pitches of the musical notes is necessitated by the idiomatic requirements of the music under study. Acceptance of musical intervals depends on cultural conditioning.

Intonation is guided by the tonotopic organization in the brain. The functional connectivity among the neural cells varies dynamically with the acoustical contexts (Weinberger, 1999, P81). The complex non-linearity in hearing along with the process of pitch perception within the context of cultural particularities culminating in the cognition of pitch in the tonotopic organization in the primary auditory cortex renders simplistic linear models unrealistic. And hence, acoustical parameters such as frequency and concordance must be replaced by their counterparts in pitch and musical consonance. A holistic approach is essential to the understanding of the musical phenomena.

6. CONCLUSION

The modern theory of intonation in South Indian *rāga* music calls for fixed frequency values for the 22 divisions of the octave, called *śruti*-s. These modern theories of pitch fixation have been attributed to Bharata for authenticity. But the ancient concept of *śruti* is not applicable to the current system of *rāga* music. The prevailing theory also places these fixed *śruti*-s among the various *rāga*-s in vogue today. Empirical pitch analysis of South Indian *rāga* music presented in this paper clearly shows that the pitching of the musical intervals varies dynamically and does not submit to the mathematical rigidities imposed by the acoustical interpretations of the modern theories of 22 *śruti*-s. South Indian *rāga* music employs flexible melodic intervals. The musical pitches are not fixed unitary quantities but they do have definite characteristic pitch profiles. The mean pitch values aren't unique either. The multiple mean pitch values demonstrate departures from the theoretically proposed frequency values. The intonation is not acoustically based and therefore transcends the acoustical parameters; intonation is psycho-physiologically driven. A holistic approach involving the dynamic process of pitch perception and the cultural context are necessary to uncover this fact.

ACKNOWLEDGEMENTS

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APPENDIX A - EXPLANATORY NOTES (for superscripted numerals)

1. The word 'Acoustic' implies determining musical intervals based on the natural harmonic series and / or on some kind of progression of cycles of fifths, fourths and thirds, leading to a mathematical formulation of musical intervals. This is a Western import. Most modern scholars have used this approach to calculate the 22 *śruti*-s elucidated by Bharata in the *Nāṭyaśāstra*. The ancients however did not employ this acoustical approach to position the 7 *svara*-s. [Abstract]

2. *Tambūra* is a fretless 4-stringed pitch instrument strum to provide the tonic and an envelope of partials to guide intonation. [Sections 1 & 5]

3. Several ancient musical treatises in Sanskrit language like the *Nāṭyaśāstra* of Bharata, *Dattilam* of Dattila, *Bṛhaddeśī* of Mataṅga and others talk about the theory of 22 *śruti*-s. According to this theory (applied to the old and now non-functional fixed-interval *grāma* system) there are 22 *śruti*-s to the octave. The intervallic arrangement in the *ṣaḍja-grāma* or the *Sa-grāma* is as follows: the third (*Ga*) and the seventh (*Ni*) notes have two *śruti*-s each, the second (*Ri*) and the sixth (*Dha*) notes have three *śruti*-s each and the first (tonic - *Sa*), the fourth (*Ma*) and the fifth (*Pa*) notes have four *śruti*-s each in them, totaling 22 *śruti*-s. Changing the *grāma* would change this intervallic arrangement. For instance, in the *madhyama-grāma* or the *Ma-grāma* the fifth note becomes a three-*śruti*-ed interval and the sixth note a four-*śruti*-ed interval. [Section 2]

4. Modern scholars such as K.B. Deval, E. Clements, F. Strangways, Abraham Pandither, B.C. Deva, Alain Daniélou, G.H. Ranade, F. Framjee, C.S. Ayyar, H.V. Modak, S. Ramanathan, P. Sambamurthy and several others are votaries of the 22-*śruti* theory; noted exceptions are N. Jairazbhoy, M. Levy, H. Powers, N. Ramanathan etc. (Komaragiri, 2005: P69) [Section 2]

5. Veṅkaṭamakhī was the celebrated author of the musical treatise, *Caturdaṇḍiprakāśikā*, written in the seventeenth century. [Section 2]

6. In the South Indian system of music, the 12 notes are conceived as 16 notes. The four additional notes are enharmonic. For example, the minor third is also taken as the enharmonic equivalent of the augmented major second. [Section 2]

7. Tonotopic organization is the characteristic pitch mapping in the primary auditory cortex (brain). These pitch maps actually alter upon deliberate learning and training. This guides the intonation. [Section 3]

8. Cent is a unit of octave measurement. It divides each semitone into 100 equal parts and the octave into 1200 equal parts. This was introduced by Englishman, J. Ellis in the 19th century in the context of equal temperament. One cent, $C = 2^{1/1200} = \sqrt[1200]{2}$ and therefore N cents = 2

$^{N/1200}$. To find the number of cents, N, in any frequency ratio, R, the formula to use is 3986 * log R to the base 10 [i.e., C = (1200/log 2) * log R]. For example, cents of the fifth note, $Pa = 3986 * \log (3/2) \approx 702$ cents. Working with cents enables multiplication to be converted into addition for easy comparison. [Section 4]

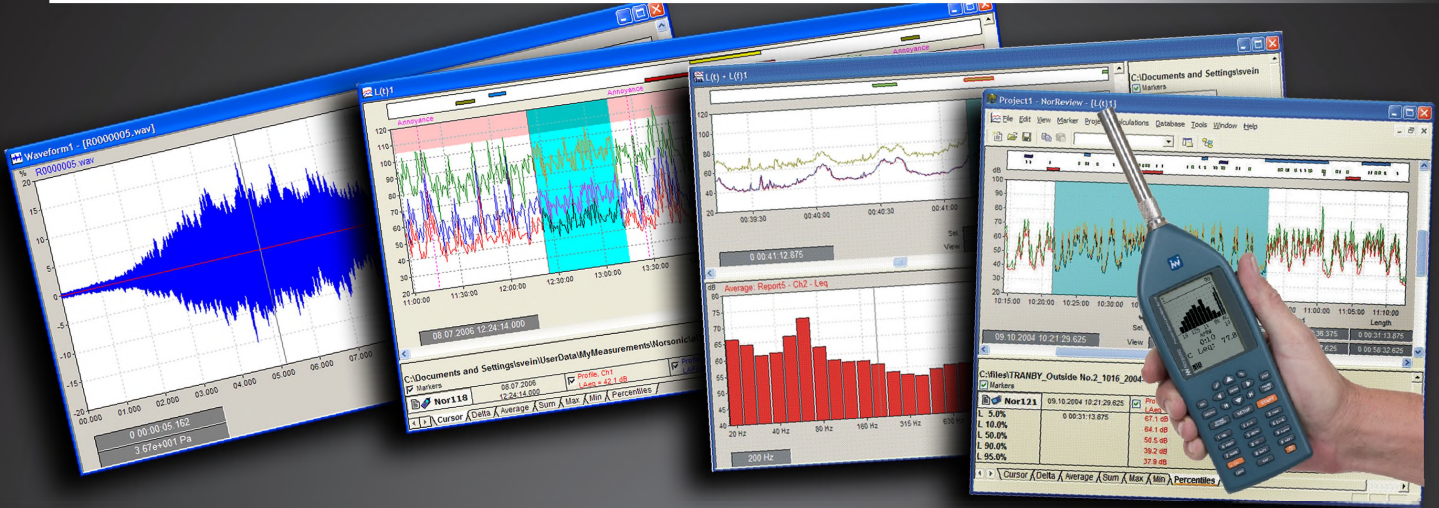9. Young and Helmholtz law states that if a string is plucked at a point of *aliquot division*, the harmonics having a *node* at the point of excitation should be entirely absent. But the *tambūra* defies this law according to C. V. Raman (1921, P471). *Aliquot* means forming an exact proper divisor, for example: an aliquot part of 15 is 5. And, *node* is a point where the amplitude of a (standing) wave is zero, i.e. a point where there is no vibration. [Section 5]

APPENDIX B - Table of Modern *Śruti* Values with *Sa* = 1

These are the commonly accepted *śruti* (frequency) values given by the modern scholars [4]. These figures are collated from various authors (Komaragiri, 2005: Appendix C). There are many other values. Sathyanarayana (1970: P70) says that there are about 250 values suggested so far.

Scholars tried to hypothesis the existence of these frequencies among the different *rāga*-s. The horizontal lines in the pitch plots given in this paper indicate these expected values. Typically, the two varieties of a *svara* are separated by 22 cents. All the cent values are rounded off.

Svara (Note)	Symbol	The 22 *Śruti* Divisions	Frequency	Cents
ṛsabha	*Ri* [Second Note]	*śuddha ṛsabha*-1	256/243	90
		śuddha ṛsabha-2	16/15	112
		catuḥ śruti ṛsabha-1	10/9	182
		catuḥ śruti ṛsabha-2	9/8	204
gāndhāra	*Ga* [Third Note]	*sādhāraṇa gāndhāra*-1	32/27	294
		sādhāraṇa gāndhāra-2	6/5	316
		antara gāndhāra-1	5/4	386
		antara gāndhāra-2	81/64	408
madhyama	*Ma* [Fourth Note]	*śuddha madhyama*-1	4/3	498
		śuddha madhyama-2	27/20	520
		prati madhyama-1	45/32	590
		prati madhyama-2	64/45	610
pañcama	*Pa* [Fifth Note]	*Pañcama*	3/2	702
dhaivata	*Dha* [Sixth Note]	*śuddha dhaivata*-1	128/81	792
		śuddha dhaivata-2	8/5	814
		catuḥ śruti dhaivata-1	5/3	884
		catuḥ śruti dhaivata-2	27/16	906
niṣāda	*Ni* [Seventh Note]	*kaiśika niṣāda*-1	16/9	996
		kaiśika niṣāda-2	9/5	1018
		kākalī niṣāda-1	15/8	1088
		kākalī niṣāda-2	243/128	1110
ṣaḍja	*Sa* [Tonic]	*tāra ṣaḍja*	2/1	1200