EXTENDING THE CAPABILITY OF THE COMPLEX EFFECTIVE DEPTH APPROXIMATION

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1. INTRODUCTION

The Pekeris waveguide, comprising a homogeneous water layer of sound speed c_w and density ρ_w above a homogeneous fluid half-space (usually of greater sound speed and density), provides a canonical configuration for studying low-frequency sound propagation in shallow water. Numerical results for the pressure in the upper layer due to a water-borne harmonic point source are readily obtained using an acoustic propagation code derived from one of the standard representations for the field, e.g., wavenumber integration, normal mode, multipath expansion, or parabolic equation [1]. Although the Pekeris waveguide represents an idealized description of a shallow water environment, it is important conceptually as it exhibits several features that are characteristic of normal mode propagation. Even for this simple configuration, however, the normal mode wave-numbers satisfy a complicated dispersion relation. As a result, they must be determined numerically using root-finding procedures. A detailed numerical analysis of a modal solution to the Pekeris waveguide for the case of a lossy fluid bottom was recently presented by Buckingham and Giddens [2].

In contrast to this exact modal approach, the "effective depth" approximation for the Pekeris waveguide, introduced by Weston [3], replaces reflection from a lossless fluid half-space by reflection from a free surface located at an appropriate distance below the actual bottom. This paradigm has undergone generalizations in recent years e.g., [4]–[7]. The effective-depth approach was modified by Chapman *et al.* [4] to derive approximate modal wavenumbers in the case of a lower half-space that supports shear. Later, Balasubramanian and Muni [5] showed that the exact modal wavenumbers could be obtained by iterating the effective-depth equations mode by mode. All of the above work ignored energy loss on reflection so that the features of the effective reflecting surface depended only on the phase of the bottom reflection coefficient. This limitation was overcome by Zhang and Tindle [6] who used the full reflection coefficient to account for energy loss on reflection. In this case, the resulting effective depth of the perfect reflector becomes complex to account for this sea-bottom loss. Subsequently, Tindle and Zhang [7] used an approximate normalization of the modal sum, based on the complex effective-depth method, to avoid sudden jumps in the behaviour of the field during modal cutoff that occurs for upslope adiabatic propagation over a sloping elastic bottom.

In this paper, we extend the effective-depth method to include two boundary features that can effect propagation in shallow water. First, we show how the coherent scattering effects due to a rough sea-surface can be accommodated within the context of the Kirchhoff approximation. Second, we demonstrate how the basic equations can be readily modified to take into account a layered ocean bottom structure. As a result, the complex effective-depth approach can be applied without difficulty to a more general class of shallow water propagation environments. Following a brief summary of the necessary equations and iteration algorithm, we present an example that exhibits the effects on normal mode propagation for a Pekeris type waveguide having a layered ocean bottom and a rough surface.

2. BASIC THEORY

For acoustic propagation in a shallow water isovelocity waveguide of depth H, the horizontal wavenumber $\boldsymbol{k}$ of each normal mode satisfies the eigenvalue condition,

$$\left[1 - R_s(k)R_b(k)\exp(2i\gamma H)\right]_{k=k_n} = 0 \ . \qquad (1)$$

Here $R_s(k)$ is the plane wave reflection coefficient at the sea surface, $R_b(k)$ is the plane wave reflection coefficient at the sea bottom, and $\gamma = (\omega^2/c_w^2 - k^2)^{1/2}$ is the vertical wavenumber corresponding to $\boldsymbol{k}$. For a flat pressure-release surface, $\boldsymbol{R_s} = -1$. To accommodate coherent scattering from a rough pressure-release surface characterized by an rms roughness, $\boldsymbol{\sigma}$, we use the Kirchhoff approximation, $\boldsymbol{R_s} = -\exp(-2\sigma^2\gamma^2)$, to write Eq. (1) in the form

$$\left[1 + \Re(k)\exp(2i\gamma H)\right]_{k=k_n} = 0 \ , \qquad (2)$$

where we have set $\Re(k) = R_b(k)\exp(-2\sigma^2\gamma^2)$. Defining the complex phase of $\Re$ via $\boldsymbol{\psi(k) = -i\ln[\Re(k)]}$ leads to an equivalent form of Eq. (2), namely

$$2\gamma(k)H + \psi(k) - \pi = 2(n-1)\pi \ . \qquad (3)$$

At this point we remark that $\boldsymbol{R_b}$ can be generalized to take into account the effects of multiple uniform layers atop a basement half-space by making use of the recursion formulas described in [1]. The complex effective-depth approximation is obtained by defining

$$\Delta H(k) = [\psi(k) + \pi]/2\gamma(k) \qquad (4)$$

and by using Eq. (4) to write Eq. (3) as

$$\gamma(k) = n\pi/[H + \Delta H] \ . \qquad (5)$$

Eq. (5) is recognized as the eigenvalue equation for the vertical wavenumber of a normal mode in an ideal isovelocity waveguide of complex depth $H+\Delta H$. Exact modal eigenvalues to the original penetrable waveguide can be obtained iteratively as follows [5],[6]: for each n, assume an initial value for k_n, use Eq. (4) to find ΔH, then Eq. (5) to find γ, and finally use $k=(\omega^2/c_w^2-\gamma^2)^{1/2}$ to obtain an improved value of k_n. The process is repeated until the value of k_n converges to a given tolerance. It is worthwhile remarking that this complex effective-depth representation allows for the treatment of both trapped and leaky modes. The acoustic pressure in the isovelocity water due to a point source at depth h is then given by

$$p(r,z) = 2i\pi\sum_{n}[H+\Delta H(k_n)]^{-1}\sin(\gamma_n h)\sin(\gamma_n z)H_0^{(1)}(k_n r) \cdot \quad (6)$$

The factor $2/[H+\Delta H(k_n)]$ is the approximate normalization for the n^{th} normal mode in an ideal waveguide with a sea-bottom at its appropriate complex effective depth.

3. EXAMPLE

To illustrate our extensions to the complex effective-depth method, we consider the shallow-water Pekeris type waveguide depicted in Fig. 1. For the modal calculations, the sediment region is modelled by a stack of 50 uniform layers each 2-m thick whose sound speeds track the gradient there. Transmission losses (TLs) vs range at 100 Hz are computed between a source and receiver at mid-depth in the water column for both flat surface ($\sigma=0$ m) and rough surface ($\sigma=2$ m) conditions. The modal TLs (ZTmode) are compared against the TLs computed using the benchmark wavenumber integration code SAFARI [8]. For the SAFARI results, 4 sublayers were used to approximate the linear variation of sound speed in the sediment region.

The transmission loss comparisons are shown in Fig. 2 for the flat surface scenario and in Fig. 3 for the rough surface scenario. The SAFARI results (shown as triangular points) have been subsampled for clarity. It is observed that the agreement between the two numerical approaches is excellent. The effects of surface roughness are seen to increase the TL due to stripping of the higher-order modes.

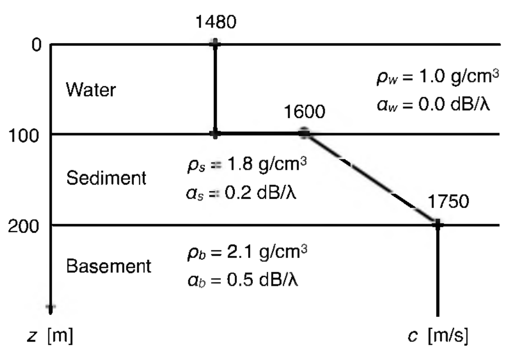

Figure 1. Pekeris type waveguide with a sediment layer.

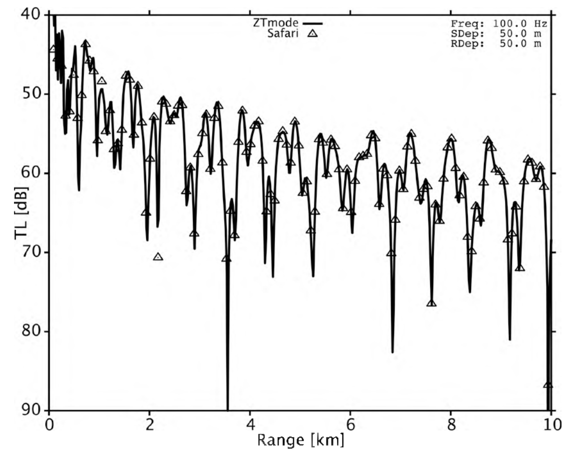

Figure 2. TL comparison for a flat surface.

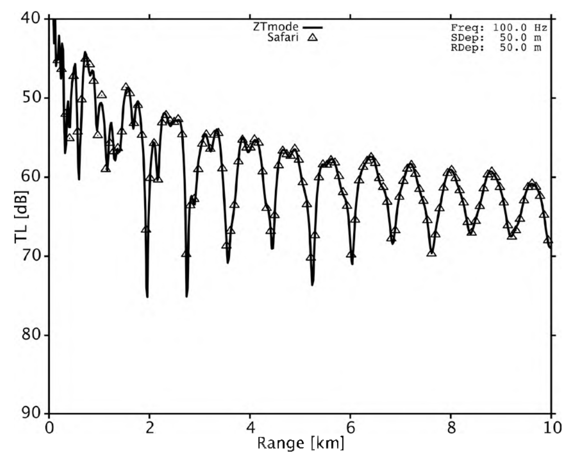

Figure 3. TL comparison for a 2-m rms rough surface.

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