THEORETICAL ANALYSIS OF LINED-DUCT SOUND ATTENUATION

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ABSTRACT

This paper investigates theoretically how duct geometry and liner thickness affect the attenuation of fundamental-mode sound propagation in a lined duct. The study was done to satisfy the need for a greater understanding of interior natural-ventilation openings and of silencers implemented to improve the acoustical performance ('ventilators'), and to provide engineers and architects with optimal-design guidelines. It assumed ventilators of the simplest form – straight, acoustically-lined ducts of rectangular cross-section. An analytical solution is presented for the attenuation of the fundamental mode in such a duct. Duct-liner thickness does not affect high-frequency performance; however, it limits low-frequency performance. A 25-mm liner is likely not thick enough to be effective, but a 100-mm liner may be excessive. Increasing the duct height reduces the attenuation at all frequencies; in order to provide effective attenuation through the 4000-Hz band, the height should not exceed 100 mm. If the cross-sectional aspect ratio of a duct is greater than 10, or the duct is only lined on two opposing surfaces, the attenuation of its fundamental mode is in effect identical to that of a 2D lined duct. Provided that the duct liner and height are such that the silencer is effective at absorbing sound at a given frequency, reducing the aspect ratio towards unity will result in large attenuation gains.

RÉSUMÉ

Cet article étudie l'influence théorique de la géométrie d'un conduit et de l'épaisseur d'un revêtement acoustique sur l'atténuation acoustique du mode fondamental. L'étude vise une meilleure compréhension des ouvertures dans les cloisons internes et des silencieux conçus pour améliorer la performance acoustique, afin d'informer les ingénieurs et les architectes des conceptions optimales. Elle fait l'hypothèse de silencieux de formes simples: un conduit rectangulaire avec un revêtement acoustique interne. Une solution analytique est présentée pour l'atténuation du mode fondamental de ce type de conduit. L'épaisseur du revêtement n'influence pas sur les performances à hautes fréquences; cependant, elle limite celles à basses fréquences. Un revêtement d'une épaisseur de 25 mm n'est pas efficace, mais 100 mm peut être excessif. Augmenter la hauteur du conduit réduit l'atténuation pour toutes les fréquences; dans le but d'obtenir une atténuation efficace aux fréquences supérieures à 4000 Hz, la hauteur ne devrait pas dépasser 100 mm. Si le rapport des dimensions latérales du conduit est supérieur à 10, ou si seulement deux surfaces opposées portent un revêtement, l'atténuation du mode fondamental est égale à celle d'un conduit 2D. Tant que le revêtement et les dimensions du conduit sont tels que le silencieux 2D absorbe efficacement le son à une fréquence particulière, une réduction du rapport résultera en une atténuation plus importante.

1. INTRODUCTION

This paper investigates theoretically how duct geometry and liner thickness affect the attenuation of fundamental-mode sound propagation in a lined duct. While the results are generally applicable, the work was done as part of larger study [1] to satisfy the need for a greater understanding of interior natural-ventilation openings and of silencers implemented to improve the acoustical performance ('ventilators'), and to provide engineers and architects with optimal-design guidelines.

Natural ventilation is increasingly employed to make buildings more sustainable [2]. It works by using wind- or buoyancy-induced pressure differentials (stack effect) to drive ventilation air through a building. Typically these pressures are small compared to those available in a mechanically-ventilated building. In order for this low pressure to drive a sufficient volume of air, it is necessary to have low airflow resistance throughout the building. To achieve this, large openings are created in internal partitions, which prove detrimental to the noise isolation between the spaces. There is a clear need for a greater understanding of interior natural-ventilation openings and of silencers implemented to improve their acoustical performance, in order to provide engineers and architects with optimal design techniques.

This paper assumes silencers of the simplest form – a straight, lined duct of rectangular cross-section. An analytical solution exists for the attenuation of the fundamental model in such a duct [3, 4]. In straight sections of lined silencers, the attenuation of the fundamental mode generally governs the performance, because it is the least attenuated [4]. This paper presents the analytical solution and uses it to investigate the effect of silencer geometry on the resulting attenuation.

2. GENERAL CARTESIAN SOLUTION

In rectangular ducts, since the geometries are made of planes defined by simple Cartesian coordinates, it is useful to use the wave equation in Cartesian coordinates. The linear wave equation for sound pressure *p* can be written as:

$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

where *c* is the sound speed. Using separation of variables to find solutions, the pressure can be expressed as the product of three spatially-dependent functions and a time-dependent function:

$$p(r,t) = p_x(x)p_y(y)p_z(z)T(t)$$

By inserting this assumption into the wave equation, the spatially-dependent variables can be separated from the time-dependent variable, creating multiple ordinary differential equations from the single partial differential equation:

$$ODEs: \begin{cases} p_x''(x) - s_x \dfrac{p_x(x)}{c^2} = 0 \\ p_y''(t) - s_y \dfrac{p_y(y)}{c^2} = 0 \\ p_z''(z) - s_z \dfrac{p_z(z)}{c^2} = 0 \\ p_t''(t) - s p_t(t) = 0 \end{cases}$$

where $s_x + s_y + s_z = s$. Differential equations of this form can take the following solutions:

$$p_{x,y\,or\,z}(x, y\,or\,z) = \begin{cases} A_1 e^{\sqrt{\frac{s_x}{c^2}}x} + A_2 e^{-\sqrt{\frac{s_x}{c^2}}x} & if\ s_x > 0 \\ A_1 + A_2 x \quad if\ s_x = 0 \\ A_1 e^{j\sqrt{\frac{-s_x}{c^2}}x} + A_2 e^{-j\sqrt{\frac{-s_x}{c^2}}x} & if\ s_x < 0 \end{cases}$$

$$p_t(t) = \begin{cases} A_1 e^{\sqrt{s}t} + A_2 e^{-\sqrt{s}t} & if\ s_t > 0 \\ A_1 + A_2 t \quad if\ s_t = 0 \\ A_1 e^{j\sqrt{-s}t} + A_2 e^{-j\sqrt{-s}t} & if\ s_t < 0 \end{cases}$$

Our interest is in the harmonic solution, for which $s_{x,\,t} < 0$. It is convenient and informative to introduce the wave number *k* and angular frequency *ω*. Letting $-s_x = k_x^2 c^2$ and $-s = \omega^2$, the general Cartesian solution can be written as:

$$p = p_x(x)p_y(y)p_z(z)p_t(t)$$

$$\text{where}: \begin{cases} p_x(x) = A_1 e^{jk_x x} + A_2 e^{-jk_x x} \\ p_y(y) = A_3 e^{jk_y y} + A_4 e^{-jk_y y} \\ p_z(z) = A_5 e^{jk_z z} + A_6 e^{-jk_z z} \\ p_t(t) = A_7 e^{j\omega t} + A_8 e^{-j\omega t} \\ k_x^{\,2} + k_y^{\,2} + k_z^{\,2} = k^2 = \dfrac{\omega^2}{c^2} \end{cases}$$

This solution represents waves, with some amplitudes and wave numbers, propagating in the positive and negative directions along each axis, and propagating in both directions with respect to time.

3. RIGID-WALLED DUCT

In an infinite-length duct, or equivalently in a duct with an anechoic termination, waves will not propagate in the –*z* direction. Waves travel forward with unit amplitude as time increases. With these restrictions, the general solution becomes:

$$p_z(z) = A_5 e^{-jk_z z}$$
$$p_t(t) = e^{j\omega t}$$

Taking the cross-section of the duct to extend from 0 to L_x in *x*, and 0 to L_y in *y*, the Neumann condition is applied to the rigid duct walls:

$$\frac{dp_x(x)}{dx} = 0 \quad at\ x = 0\ and\ x = L_x$$
$$\frac{dp_y(y)}{dy} = 0 \quad at\ y = 0\ and\ y = L_y$$

Using these boundary conditions, and the general solution, a modal solution can be presented as:

$$p = A\cos(k_l x)\cos(k_m y)e^{j(\omega t - k_z z)}$$
$$k_l^{\,2} + k_m^{\,2} + k_z^{\,2} = \frac{\omega^2}{c^2}$$
$$where \begin{cases} k_l = \dfrac{l\pi}{L_x} & l = 0,1,2,3\ldots \\ k_m = \dfrac{m\pi}{L_y} & m = 0,1,2,3\ldots \end{cases}$$

By letting $k_l^2 + k_m^2 = k_{lm}^2$, and solving for the wave number in *z*, some properties of the system become apparent:

$$k_z = \sqrt{\frac{\omega^2}{c^2} - k_{lm}^{\,2}}$$

For relatively high frequencies, or ducts of large cross-section with respect to a given mode, the pressure fluctuates sinusoidally with z:

$$\frac{\omega^2}{c^2} - k_{lm}{}^2 > 0$$

$$p = A\cos(k_l x)\cos(k_m y)e^{j\omega t}e^{-j\sqrt{\frac{\omega^2}{c^2} - k_{lm}{}^2}z}$$

When the frequency becomes low, or the duct is small with respect to a given mode, the wave number becomes complex, resulting in a pressure that decays exponentially with increasing z. This defines the cut-off frequency for a mode in a duct. The only mode that does not have a cut-off frequency is the plane-wave mode ($l = 0$, $m = 0$):

$$\frac{\omega^2}{c^2} - k_{lm}{}^2 < 0$$

$$p = A\cos(k_l x)\cos(k_m y)e^{j\omega t}e^{-\sqrt{k_{lm}{}^2 - \frac{\omega^2}{c^2}}z}$$

4. NON-RIGID-WALLED DUCT

If a duct does not have rigid walls, the Neumann boundary condition becomes invalid. If the normal-incidence surface impedance is known, then the boundary condition can be replaced with:

$$Z_s = \frac{p}{u}$$

Applying Newton's second law to an element of fluid, the particle velocity can be related to pressure:

$$F = m\ddot{x}$$
$$\frac{\partial p_x}{\partial x} = -\rho_0 \frac{\partial u_x}{\partial t}$$
$$\frac{\partial p_y}{\partial y} = -\rho_0 \frac{\partial u_y}{\partial t}$$

Assuming that u_x and u_y have solutions that vary sinusoidally with time, it follows that:

$$u_x = \frac{-1}{j\rho_0\omega}\frac{dp_x}{dx} = \frac{-k_x}{\rho_0 kc}\left(A_1 e^{jk_x x} - A_2 e^{-jk_x x}\right)$$
$$u_y = \frac{-1}{j\rho_0\omega}\frac{dp_y}{dy} = \frac{-k_y}{\rho_0 kc}\left(A_3 e^{jk_y y} - A_4 e^{-jk_y y}\right)$$

Solving for the impedance at the duct walls (h_x, $-h_x$, h_y, $-h_y$):

$$Z_{s,x}(h_x) = \frac{A_1 e^{jk_x h_x} + A_2 e^{-jh_x}}{\frac{-k_x}{\rho_0 kc}\left(A_1 e^{jh_x} - A_2 e^{-jh_x}\right)}$$
$$Z_{s,x}(-h_x) = \frac{A_1 e^{-jk_x h_x} + A_2 e^{jh_x}}{\frac{-k_x}{\rho_0 kc}\left(A_1 e^{-jh_x} - A_2 e^{jh_x}\right)}$$
$$Z_{s,y}(h_y) = \frac{A_3 e^{jk_y h_y} + A_4 e^{-jk_y h_y}}{\frac{-k_y}{\rho_0 kc}\left(A_3 e^{jk_y h_y} - A_4 e^{-jk_y h_y}\right)}$$
$$Z_{s,y}(-h_y) = \frac{A_3 e^{-jk_y h_y} + A_4 e^{jk_y h_y}}{\frac{-k_y}{\rho_0 kc}\left(A_3 e^{-jk_y h_y} - A_4 e^{jk_y h_y}\right)}$$

If the impedances of opposite walls are equal, the simplifying assumption can be made that the propagating modes will be either symmetric or antisymmetric [3]. For symmetric-mode propagation:

$$A_1 = A_2$$
$$A_3 = A_4$$
$$\frac{Z_{s,x}(h_x)}{Z_0} = \frac{-Z_{s,x}(-h_x)}{Z_0} = \frac{-k}{jk_x}\cot(k_x h_x)$$
$$\frac{Z_{s,y}(h_y)}{Z_0} = \frac{-Z_{s,y}(-h_y)}{Z_0} = \frac{-k}{jk_y}\cot(k_y h_y)$$

For antisymmetric-mode propagation:

$$-A_1 = A_2$$
$$-A_3 = A_4$$
$$\frac{Z_{s,x}(h_x)}{Z_0} = \frac{-Z_{s,x}(-h_x)}{Z_0} = \frac{k}{jk_x}\tan(k_x h_x)$$
$$\frac{Z_{s,y}(h_y)}{Z_0} = \frac{-Z_{s,y}(-h_y)}{Z_0} = \frac{k}{jk_y}\tan(k_y h_y)$$

Re-written, the system of equations for a duct in which opposite walls have the same impedance is:

$$\left.\begin{aligned} k_x \tan(k_x h_x) &= \frac{jkZ_0}{Z_{s,x}} \\ k_y \tan(k_y h_y) &= \frac{jkZ_0}{Z_{s,y}} \\ k_x{}^2 + k_y{}^2 + k_z{}^2 &= k^2 \end{aligned}\right\} \text{symmetric}$$
$$\left.\begin{aligned} k_x \cot(k_x h_x) &= \frac{-jkZ_0}{Z_{s,x}} \\ k_y \cot(k_y h_y) &= \frac{-jkZ_0}{Z_{s,y}} \\ k_x{}^2 + k_y{}^2 + k_z{}^2 &= k^2 \end{aligned}\right\} \text{antisymmetric} \qquad (1)$$

These two sets of equations can be solved numerically to find k_z as a function of k (k is directly related to frequency). A numerical-iteration scheme, such as the Newton-Raphson method, can be used to find the roots of, and solutions to, these equations.

One important observation from this analysis is that, if the wall impedance is not infinite, the wavenumbers will be complex. If the wavenumber in z is complex, the pressure variation p_3 with respect to the duct length can be written:

$$p_3(z) = A_5 e^{j\,\mathrm{Re}(k_z)z} e^{-\mathrm{Im}(k_z)z}$$

This result shows that the modal pressure decays exponentially along the length of the duct. The attenuation can be conveniently expressed in decibels as:

$$\text{Attenuation} = 20\,\mathrm{Im}(k_z)z\log(e) = 8.686\,\mathrm{Im}(k_z)z \text{ dB}$$

4.1 Defining the Surface Impedance

A solution for the plane-wave attenuation in a lined duct has been presented; however, the surface impedance of the absorptive liner must be known. The transfer-function method is presented here as a simple method for converting an absorptive material's characteristic impedance and wave number into a surface impedance. A brief background is also given on absorptive materials, to describe how the propagation impedance and wavenumber are determined.

4.1.1 Transfer-function method

In order to use the propagation impedance and wavenumber for design in typical applications, they must be converted into an equivalent surface impedance [5, 6]. The transfer-function method is convenient for this purpose. It starts by defining the pressure and velocity at positions $x = 0$ and $x = d$ as functions of the forward and backward propagating waves. These four equations are then rearranged to relate the pressure and velocity at $x = d$ to those at $x = 0$ by a general 'transfer function':

$$p_x(0) = A_1 + A_2$$
$$u_x(0) = \frac{(A_1 - A_2)}{\rho\omega}$$
$$p_x(d_x) = A_1 e^{-jk_x d_x} + A_2 e^{jk_x d_x}$$
$$u_x(d_x) = \frac{k_x\left(A_1 e^{-jk_x d_x} - A_2 e^{jk_x d_x}\right)}{\rho\omega}$$

Subscript x indicates the component of the variable in the x direction. Combining these equations gives:

$$p_x(0) = p(d_x)\cos(k_x d_x) + ju(d_x)\frac{\rho\omega}{k_x}\sin(k_x d_x)$$
$$u_x(0) = \frac{jk_x p(d_x)}{\rho\omega}\sin(k_x d_x) + u(d_x)\cos(k_x d_x)$$

which can be equivalently expressed in matrix form as:

$$\begin{bmatrix} p(0) \\ u(0) \end{bmatrix} = [T]\begin{bmatrix} p(d_x) \\ u(d_x) \end{bmatrix}$$

$$[T] = \begin{bmatrix} \cos(k_x d_x) + j\dfrac{\rho\omega}{k_x}\sin(k_x d_x) \\ \dfrac{jk_x}{\rho\omega}\sin(k_x d_x) + \cos(k_x d_x) \end{bmatrix}$$

$[T]$ is the transfer matrix for a finite-thickness layer. Transfer matrices can be defined for many simple geometries, and are multiplied together to find the total transfer functions of compound layers and geometries. Here, we see that, if we let the surface impedance at $x = d$ be $Z_{s,x}(d_x)$, we can solve for the surface impedance at $x = 0$, $Z_s(0)$:

$$Z_{s,x}(0) = \frac{Z_{s,x}(d_x)\cot(k_x d_x) + jZ_0\dfrac{k}{k_x}}{j\dfrac{Z_s,x(d_x)k_x}{Z_0 k} + \cot(k_x d_x)}$$

If the layer is backed by a rigid surface, then $Z_{s,x}(d_x)$ is effectively infinite and the surface impedance can be simplified to:

$$Z_{s,x}(0) = -jZ_0\frac{k}{k_x}\cot(k_x d_x) \tag{2}$$

This result can be used, in combination with Eqs. (1), to define the surface impedance of a duct, provided the propagation impedance and wavenumber, Z_0 and k respecttively, are known for the porous absorber. For clarity, from here on the characteristic impedance and wavenumber of the porous absorber are identified as Z_w and k_w. The symmetric equations are:

$$\left.\begin{aligned} -jZ_w\frac{k_w}{k_{w,x}}\cot(k_{w,x}d_x) &= j\cot(k_x h_x)\frac{jkZ_0}{k_x} \\ -jZ_w\frac{k_w}{k_{w,y}}\cot(k_{w,y}d_y) &= j\cot(k_y h_y)\frac{jkZ_0}{k_y} \\ {k_x}^2 + {k_y}^2 + {k_z}^2 &= k^2 \end{aligned}\right\} \tag{3}$$

This formulation allows for arbitrary incidence angle; however, $k_{w,x}$ must be found using Snell's law, as refraction occurs due to the difference in wave speeds in air and in a porous absorber. With ψ and φ being the incident and transmitted angles, $k_{w,x}$ is [5]:

$$k_{x,w} = k_w\sqrt{1-\sin(\phi)} = \sqrt{{k_w}^2 - k^2\sin(\psi)}$$

In practice, the wave speeds in many porous materials are much smaller than in air; thus the waves propagate nearly normal to the surface [7]. Considering this effect, $k_{w,x} \approx k_w$. Materials in which sound will only propagate normal to the surface are referred to as 'locally reacting'. The surface impedance of a rigidly-backed, locally-reacting absorber is:

$$Z_{s,x} = -jZ_w \cot(k_w d_x)$$

The local-reaction assumption is valid provided $R < 4$ [7], where R is the normalized flow resistance, given by:

$$R = \frac{\sigma d}{\rho_0 c}$$

in which σ is the flow resistivity in MKS Rayl/m.

4.1.2 Characterizing porous absorptive materials

Porous acoustical absorbers are materials that absorb sound energy passively by means of thermal dissipation. As sound waves propagate through the porous material, the shear forces due to no-slip conditions at the absorber surface convert the kinetic energy into heat. In addition, the high surface area in the porous material makes the compression process non-adiabatic.

Porous absorbers are, as shown above, most usefully described in terms of their acoustical propagation impedance and wavenumber. Many methods, both empirical and analytical, have been developed to determine the acoustical impedance, based on material properties [5, 7]. Analytical methods, based on models of the microscopic fluid domain, have proven successful; however, they are quite complicated compared to empirical methods. Empirical methods, such as the well-known Delaney-Bazley model [5], provide a simple method for calculating the impedance from easily measured properties.

The Delaney-Bazley model is based on a data curve-fit of many samples of fibrous acoustical absorbers with different flow resistivities; therefore, it should not be expected to give accurate results for non-fibrous absorbers, such as open-cell foams. The acoustical impedance and wavenumber of a fibrous porous absorber are [5]:

$$\frac{Z_w}{Z_0} = 1 + 0.0571X^{-0.754} - j0.087X^{-0.732}$$

$$k_w = \frac{\omega}{c_0}\left(1 + 0.0978X^{-0.700} - j0.189X^{-0..595}\right)$$

X is a function of the flow resistivity σ and frequency f:

$$X = \frac{\rho_0 f}{\sigma}$$

The Delaney-Bazley model is a single log-linear curve fit of the real and imaginary components of the impedance and wavenumber, to represent all fibrous absorbers. It is valid when [5]:

- ε (porosity) ≈ 1
- $0.01 < X < 1.0$
- $1000 < \sigma < 50{,}000$ MKS Rayl/m.

5. RESULTS

To investigation plane-wave attenuation in a lined duct, it is necessary to define realistic liner properties. For this analysis, a liner was defined to have properties similar to the material used in laboratory-measured cross-talk silencers [2]. Once a lining material was established, the effect of geometry on attenuation was investigated.

5.1 Duct-Liner Properties

To use the Delaney-Bazley method of describing the porous material, it is necessary to define the material's flow resistivity. This was done by selecting a flow resistivity that, using the Delaney-Bazley model and transfer-function methods, defines a material with a similar normal-incidence absorption coefficient a to that of the liner used in laboratory measurements [1]. Using the pressure-reflection coefficient r, the normal-incidence coefficient can be calculated from the surface impedance:

$$a = 1 - |r|^2$$

$$r = \frac{Z_s - \rho_0 c}{Z_s + \rho_0 c}$$

The normal-incidence absorption coefficient of a 25-mm-thick OEM glass-fiber sample was measured using an impedance tube and a standardized measurement procedure [8]. A comparison between the measured glass-fiber material and Delaney-Bazley prediction for different flow resistivities is shown in Figure 1. Data above 2000 Hz could not be obtained, due to impedance-tube limitations.

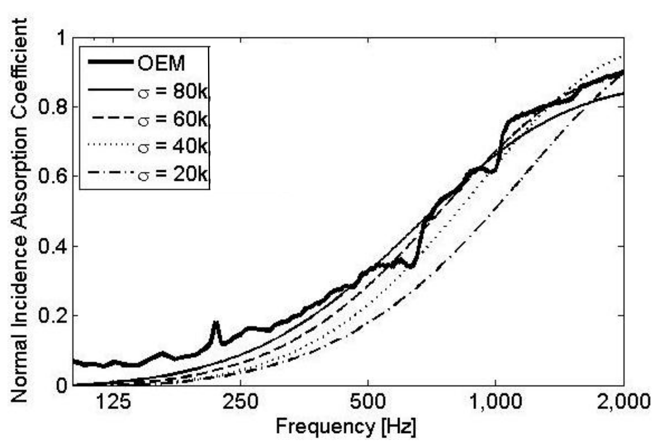

Figure 1: Absorption coefficient of 25-mm-thick OEM glass fiber as measured, and as predicted by the Delaney-Bazley model for different flow resistivities, σ in MKS Rayl/m.

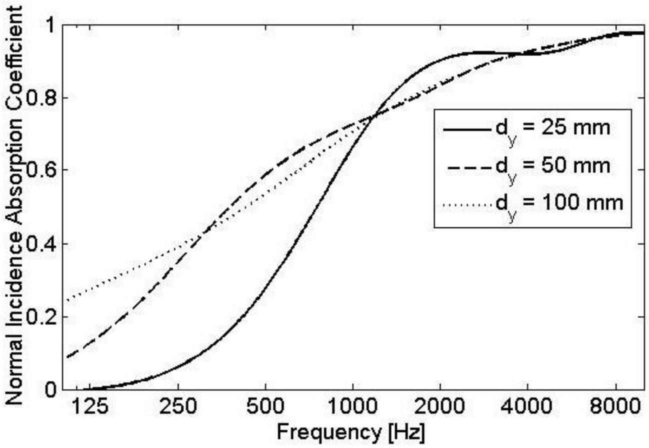

Figure 2: Variation of normal-incidence absorption coefficient for various liner thicknesses as predicted by the Delaney-Bazley model with σ = 60,000 MKS Rayl/m.

Above 500 Hz, the predicted absorption agrees best with the measurement when the flow resistivity is 60,000 MKS Rayl/m; however, below 500 Hz the Delaney-Bazley model under-predicts the measured absorption. Using a higher flow resistivity would slightly increase the low-frequency absorption; however, it would step outside of the range of validity of the local-reaction assumption. Direct measurements of the OEM glass fiber showed the flow resistivity to be 46,000 MKS Rayl/m [9]. In summary, reasonable normal-incidence-absorption agreement occurs for σ = 60k MKS Rayl/m.

The absorption is also strongly dependent on the liner thickness. Using a material with a flow resistivity of 60,000 MKS Rayl/m, the Delaney-Bazley model was used to calculate the absorption coefficient of a layer of glass fiber with varying thickness. The results are shown in Figure 2. All liner thicknesses generally provide increased absorption with increasing frequency. Above 1 kHz, all three liners have high absorption. Decreasing liner thickness results in decreased absorption at low frequency. The 25-mm liner is effectively incapable of absorbing in the 125-Hz octave band; only modest absorption is achieved in the 125-Hz band with a 100-mm liner.

5.2 Cross-Sectional Dimensions

To optimize the performance of a straight section of lined duct, one must consider the effect of the silencer flow-path dimensions, lining thickness and the acoustical proper-

Figure 3: Silencer dimensions.

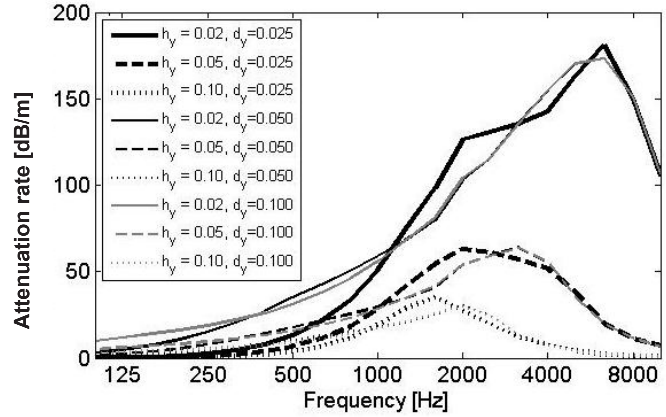

Figure 4: Predicted attenuation rate for various duct heights (h_y) and liner thicknesses (d_y).

ties of the liner. The height and width of the flow cavity in the silencer both have great effects on the acoustical attenuation; the cross-sectional geometry was examined by looking at the effects of flow-path height and aspect ratio, and how the behaviour depended on liner thickness. As required for Eqs. (3), the silencer height was equal to $2h_y$, and the liner thickness was d_y (Figure 3). The plane-wave attenuation was determined by solving Eqs. (1) and (2) using the Newton-Raphson numerical-iteration scheme.

5.2.1 Flow-path height

To examine the effect of flow-path height, a 2D silencer was studied. Attenuation of the fundamental mode in a 2D silencer is identical to that in a 3D silencer with: a. the same height, and with width much larger than the height; b. the same height and any width, but lined only on the top and bottom surfaces.

Figure 4 shows the attenuation rate in dB/m of the first-order mode in a duct with varying height and absorber thickness, plotted against frequency. If the attenuation rate (already a logarithm of power) is plotted on a logarithmic

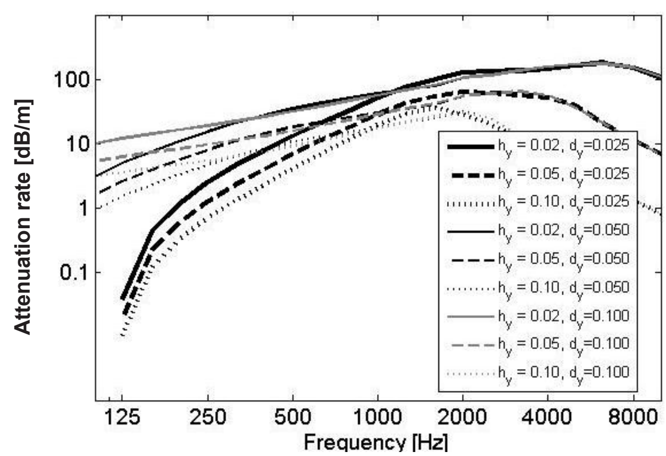

Figure 5: Predicted attenuation rate for various duct heights (h_y) and liner thicknesses (d_y), with attenuation rate plotted on a logarithmic scale.

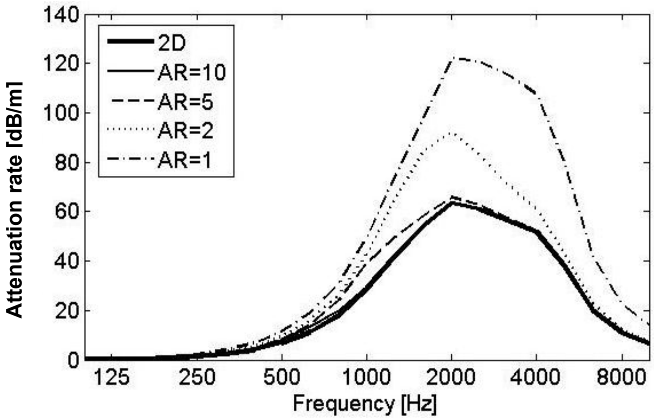

Figure 6: Predicted attenuation rate for various duct aspect ratios (AR): h_y=50 mm, h_x=h_y·AR, d_y=d_x=25 mm.

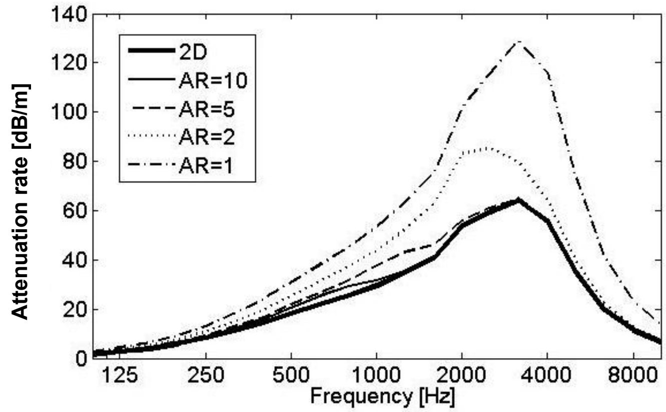

Figure 7: Predicted attenuation rate for various aspect ratios (AR): h_y=50 mm, h_x=h_y·AR, d_y=d_x=50 mm.

scale with respect to frequency, the relationships are better illustrated (see Figure 5).

It is apparent that, at low frequencies, the attenuation rate is governed by the absorber thickness. Below 1000 Hz the performance of the silencer with a 25-mm liner falls off relative to those of the 50- and 100-mm liners. Likewise, below 250 Hz the attenuation with 50-mm liner falls off with respect to that of the 100-mm-thick liner. This result is consistent with the normal-incidence absorption-coefficient results shown in Figure 2.

Above 250 Hz, for the 50-mm liner, and above 1000 Hz for the 25-mm liner, the attenuation rate is not governed by the thickness of the liner (although it may be affected by the flow resistivity). In this region the attenuation rate is limited by the rate at which energy in the fundamental mode diffracts into the absorptive material. In all cases, the frequency at which the attenuation is maximized is very close to the frequency at which the wavelength is equal to the duct height (2*h*).

5.2.2 Flow-path aspect ratio

In the previous section the relationship between duct height and attenuation was investigated. To calculate the fundamental-mode attenuation, a 2D duct, equivalent to a duct with infinite width or a duct only lined on two opposing surfaces, was investigated. This section investigates the effect on the attenuation of the fundamental mode of a duct of varying the aspect ratio, with all four walls acoustically lined. Figure 6 shows the effect of varying the aspect ratio of a lined duct with a 0.1-m total internal height, and all four walls lined with 25-mm-thick absorptive material. As expected, if the aspect ratio is large (AR>10), the result is effectively identical to that of the 2D solution. As the aspect ratio decreases, there is an increase in attenuation. The increase in attenuation due to a reduction in AR appears to be directly related to the original attenuation – that is, if the 2D silencer has negligible attenuation, reducing the AR will not result in significant attenuation. If a 2D silencer has significant attenuation at a given frequency, a silencer with the same height, but AR = 1, will have greatly increased attenuation. Figures 7 and 8 show the same result for ducts with 50- and 100-mm-thick absorptive liners. The same results are observed for all liner thicknesses; however, as before, the attenuation rates are more pronounced at lower frequencies for thicker liners.

The increase in the attenuation of a lined duct with a small aspect ratio should be expected. With a 2D duct the wavefront will form a 2D arc as it diffracts into the liner. Because the length of an arc increases in proportion to the arc radius, the maximum energy-attenuation rate is inversely proportional to the radius. In a 3D duct with AR = 1, the wavefront will approximate the spherical end of a 3D cone as it diffracts into the liner. The area of a sphere increases in proportion to the radius squared; therefore the maximum attenuation is inversely proportional to the radius squared. As attenuation rate is expressed on a logarithmic scale, the attenuation rate in a duct with an aspect ratio of 1 is twice that in a 2D duct (or, equivalently, with AR>10) with the same height. Figures 6, 7 and 8 suggest that the attenuation rate with AR = 1 is indeed nearly twice the 2D value for any duct configuration and frequency.

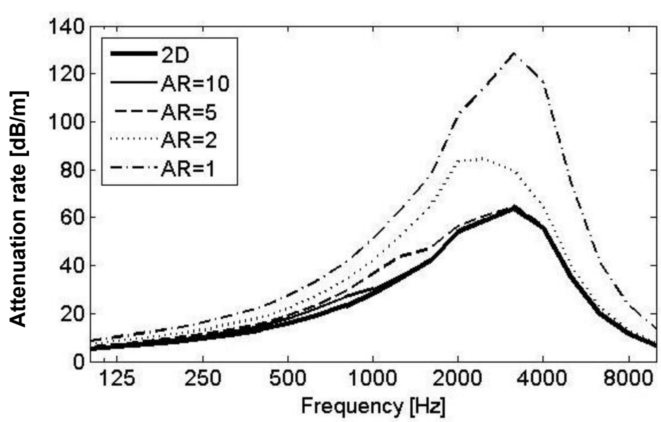

Figure 8: Predicted attenuation rate for various aspect ratios (AR): h_y=50 mm, h_x=h_y·AR, d_y=d_x=100 mm.

6. CONCLUSION

Through comparison of the absorption coefficients, it was determined that a fibrous material with a flow resistivity of 60,000 MKS Rayl/m, as defined by the Delaney-Bazley model, has similar acoustical performance to the glass-fiber liner used in laboratory measurements [1]. Using this material, with an analytical solution for plane-wave attenuation in a lined duct, the effects of varying the duct's cross-sectional dimensions have been analyzed, providing information about how liner thickness, duct height and duct aspect ratio affect attenuation.

Duct-liner thickness does not affect high-frequency performance; however, it limits low-frequency performance. The performance of a 25-mm liner falls off below 1000 Hz; that of a 50-mm liner falls off below 250 Hz. From ventilation-opening laboratory measurements [1], it was observed that the performance of natural-ventilation-opening silencers is often limited by the 500-Hz frequency band. This result was based on the assumption that the sound that natural-ventilation-opening silencers are required to attenuate is speech. As a result, a 25-mm liner is likely not thick enough to be effective; however, a 100-mm liner may be excessive. Increasing the duct height reduces attenuation at all frequencies; however, if the frequency is high enough, or the duct is large enough that the wavelength is shorter than the duct height, the attenuation decreases rapidly. In order to provide effective attenuation through the 4000-Hz band, the duct height should not exceed 100 mm. In the case of using ducts as silencers in natural-ventilation openings to control the propagation speech sounds, smaller duct heights may be more appropriate than in the case of ducts silencers controlling lower-frequency mechanical-ventilation noise.

If the aspect ratio of a duct is greater than 10, or it is only lined on two opposing surfaces, the attenuation of its fundamental mode is, in effect, identical to that of a 2D duct.

Provided the duct liner and dimensions are such that the 2D silencer is effective at absorbing sound at a given frequency frequency, reducing the aspect ratio to near unity results in large attenuation gains. The attenuation rate of a lined duct with AR=1 is approximately twice that of a 2D lined duct.

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