GRADIENT REPRESENTATIONS IN SEABED GEOACOUSTIC INVERSION BY BERNSTEIN POLYNOMIALS

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1 Introduction

Geoacoustic properties of the upper-most transition layer of mud seabed sediments often change rapidly with depth as continuous gradients, rather than discontinuous layers. However, most geoacoustic inversion approaches are based on layered sediment models. This paper presents a seabed parameterization approach that represents continuous geoacoustic gradients as a sum of Bernstein polynomial basis functions weighted by unknown coefficients which are estimated by Bayesian inversion of seabed acoustic reflectivity data. The Bernstein representation is efficient/effective in representing a wide variety of gradients with a small number of coefficients, and has optimal numerical stability to perturbation of the coefficients in the nonlinear inversion scheme. The Bernstein parametrization in geoacoustic inversion is applied to simulated data to illustrate the ability of the Bernstein polynomial parameterization to represent steep and strongly-variable gradients.

The development of acoustic remote sensing techniques for seabed parameter estimation is an active area of research, relevant to a variety of fields such as sonar signal processing, underwater noise modelling, and bioacoustics.[1,2] With a trend of increasing computational power accessible to most researchers [including parallel computing architectures and graphic processing units (GPUs)], application of sophisticated Bayesian geoacoustic inversion methods can be used to exploit the information content of acoustic data to reveal fine-scale seabed layering structure, and to estimate sediment sound speeds, densities, attenuations, and parameter uncertainties.[3,4] A key component in geoacoustic inversion is the efficient representation of the unknown media under study (i.e., the seabed) by a proposed mathematical model using only a few parameters. The majority of works on seabed geoacoustics have focused on layered models [Fig. 1(a)], appropriate to represent seabeds with sharp depth-dependent transitions between different kinds of sediment as a function of depth. However, such representation is highly inefficient in cases of smooth depth-dependent variations [Fig. 1(b)], such as those observed in the sound speed and density of mud sediments. This work proposes the use of Bernstein polynomials as the natural choice to represent continuous sediment properties.

The acoustic data considered in this work are the

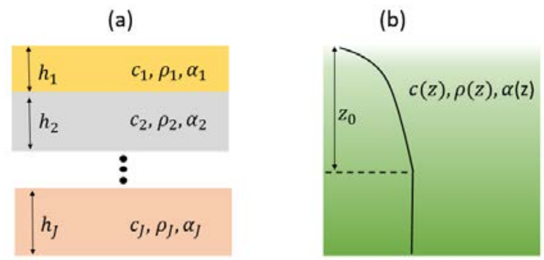

Figure 1: Representations of depth-dependent variation of seabed parameters: (a) discrete layering with constant geoacoustic properties within each layer and (b) continuous variation at $0 \le z \le z_o$.

seabed bottom loss (BL) defined as[1,2]

$$BL(\theta, f) = -10\log_{10}|R(\theta, f)| \quad (1)$$

where θ is the grazing angle, f is the frequency, and $R(\theta, f)$ is the seabed reflection coefficient, which in field experiments can be obtained by the wide-angle reflection technique.[5] This quantity has been shown to carry substantial information content on seabed geoacoustics at and near the angle of intromission for muddy (silty-clay) sediments. The inversion approach in this work exploits this information.

2 Method

2.1 Gradient Bernstein representation

Let $c(z)$ and $\rho(z)$ be continuous functions representing the (smooth) depth-dependent variation of the sound speed and density in the seabed, respectively, as illustrated in Fig.1(b). Their polynomial representations in Bernstein form are[6]

$$\begin{aligned} c(z) &= \textstyle\sum_{j=0}^{J^c} g_i^c b_j(z, J^c), \\ \rho(z) &= \textstyle\sum_{j=0}^{J^\rho} g_i^\rho b_j(z, J^\rho), \end{aligned} \quad (2)$$

where J^c and J^ρ are the order of the polynomial, g_i^c and g_i^ρ are real coefficients determined by Bayesian inversion, and[6]

$$b_j(z, J^{c,\rho}) = \binom{J^{c,\rho}}{j} (1-z)^{J^{c,\rho}-j} z^j, \quad (3)$$

are the Bernstein basis functions for $0 \le z \le z_o$.

2.2 Bayesian geoacoustic inversion

Bayesian geoacoustic inversion methods estimate the posterior probability density (PPD) defined as[3,4]

$$P(\mathbf{m}|\mathbf{d}) = \frac{P(\mathbf{m})P(\mathbf{d}|\mathbf{m})}{Z}, \quad (4)$$

where $\mathbf{d}$ is a vector containing $BL(\theta, f)$ at N_θ angles and N_f frequencies, Z is a normalizing constant, $P(\mathbf{m})$ represents prior information on the sediment geoacoustics, the

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conditional probability $P(\mathbf{d}|\mathbf{m})$ is interpreted as the likelihood function model, and

$$\mathbf{m} = \left[z_o\ g_0^c\ g_1^c \ldots\ g_{J^c}^c\ g_0^\rho\ g_1^\rho \ldots\ g_{J^\rho}^\rho\ \alpha\right], \qquad (5)$$

is the vector used to parametrize the seabed according to the Eqs. (2) and (3), with α representing the sediment attenuation (assumed constant over depth in this study). Predicted BL data, $\mathbf{d}(\mathbf{m})$, can be obtained as the plane wave reflection coefficient (PWRC) corresponding to sediments described by $\mathbf{m}$. Assuming that data residuals $\mathbf{e} = \mathbf{d} - \mathbf{d}(\mathbf{m})$ are Gaussian distributed with covariance matrix $\mathbf{C_d}$, the likelihood is defined as[3-4]

$$P(\mathbf{d}|\mathbf{m}) = \frac{1}{(2\pi)^{N_\theta N_f/2}|\mathbf{C_d}|^{1/2}} \exp\left(-\frac{1}{2}\mathbf{e}^T\mathbf{C_d}^{-1}\mathbf{e}\right). \qquad (6)$$

In this work, PWRC predictions were implemented on a GPU for efficient computation. The PPD was quantified using Metropolis-Hastings sampling.[3,4] To provide a more complete search over the parameter space, parallel tempering is applied by running 16 interacting Markov chains at a sequence of sampling temperatures between 1 and 5, with chains at higher temperatures sampling wider regions of the parameter space.

3 Results

Simulated BL data with added zero-mean Gaussian noise (0.5 dB standard deviation) were computed for a sediment profile that mimics core measurements[7], with a steep positive depth-dependent gradient for the density and a mild negative gradient for the sound speed. For the inversion, non-informative uniform distributions with bounds given in Table 1 were assumed.

Table 1: Bounds of the uniform distributions used as priors.

Parameter	Units	min	max
z_o	m	0	1
α	dB/m/KHz	0	0.05
g_i^ρ	g/cm^3	0.5	2
g_i^c	m/s	1400	1700

Figure 2 shows marginal PPD profiles for sound speed, density, and attenuation obtained by Bayesian sampling, with $J^c = 5$ and $J^\rho = 6$. At most depths, the PPD overlaps the true sound speeds and densities, capturing some of the fine scale features of the profiles but requiring only a small number of parameters. Attempting such inversion with a layered modeled would substantially enlarge the dimensionality of the parameter space: instead of the 2+$J^c + J^\rho$ parameters used by the Bersntein representation, a layered model would require many layers with four unknowns per layer to represent a continuous profile. Inversions in large parameter spaces are computationally challenging, and are prone to introduce artifacts in the form of sharp transitions in the estimated PPDs.

4 Conclusions

This paper discusses the use of Bernstein polynomials to represent continuous variation in seabed parameters. In addition to efficient representation of smooth variations, Bernstein polynomials can be shown to exhibit optimal numerical stability when its weighting coefficients are perturbed, which is a highly advantageous property for gradual exploration of multi-dimensional parameter spaces in Bayesian sampling inversions

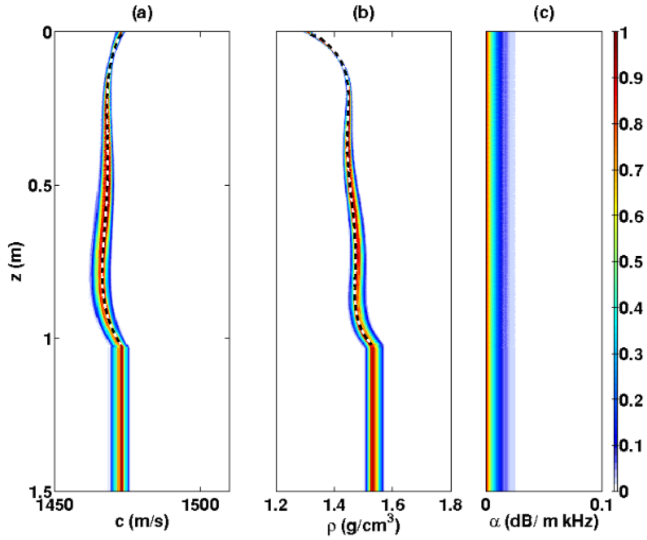

Figure 2: (a) Sound-speed, (b) density, and (c) attenuation profiles obtained by Bayesian inversion of simulated BL data. Dashed lines indicate true parameters.

References

[1] R. J. Urick, Principles of underwater sound (McGraw-Hill, Inc., New York, 1983), pp. 1-423.

[2] F. B. Jensen, W. A. Kuperman, M. B. Porter, and H. Schmidt, Computational Ocean Acoustics (American Institute of Physics, New York, 1994), pp. 1-794.

[3] J. Dettmer, C. W. Holland, and S. E. Dosso, “Analyzing lateral seabed variability with Bayesian inference of seabed reflection data”, J. Acoust. Soc. Am. 126, 56–69 (2009).

[4] G. Steininger, S. E. Dosso, C. W. Holland, and J. Dettmer, “A trans-dimensional polynomial-spline parameterization for gradient-based geoacoustic inversion”, J.Acoust. Soc. Am. 136, 1563–1573 (2014).

[5] C. W. Holland and J. Osler, “High resolution geoacoustic inversion in shallow water: a joint time- and frequency-domain technique”, J. Acoust. Soc. Am. 107, 1263–1279 (2000).

[6] R. T. Farouki, “The Bernstein polynomial basis: A centennial retrospective”, Comp. Aided Geometric Des. 29, 379–419 (2012).

[7] C. W. Holland, J. Dettmer, and S. E. Dosso, “Remote sensing of sediment density and velocity gradients in the transition layer”, J. Acoust. Soc. Am. 118, 163–177 (2005).

Acknowledgments

The authors gratefully acknowledge NSERC support. Jan Dettmer acknowledges the support of the Office of Naval Research grant N000140910394.