

# ACOUSTICAL PROPERTIES OF HOMOGENEOUS PLANAR SURFACES IN ROOM ACOUSTICS: A THEORETICAL REVIEW

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## 1 Introduction

This work reviews the fundamental boundary conditions used in room acoustics from a theoretical perspective. In particular, it is assumed that sound propagates in air as plane waves and the boundaries are planar walls. Moreover, the walls are assumed to be infinite in lateral extent and have uniform properties along their surface. These two conditions ensure that sound waves are reflected wholly specularly, as opposed to being scattered as well; the assumption of infinite lateral extent avoids scattering from the edges of the walls, and the assumption of uniform properties avoids scattered radiation caused by the inhomogeneity of the wall. Parameters used in describing the acoustical properties of the walls are introduced and, using the wave equation, expressions are derived for them in terms of the wall impedance. Theoretical relations between the parameters are then discussed based on (i) physical restrictions such as energy conservation and the causality condition and (ii) simplifying assumptions used in room acoustics such as local reaction of surfaces and the minimum-phase condition. Finally, some practical aspects of these theoretical relations are discussed using simple examples.

## 2 Acoustical Descriptors of Room Surfaces

The effects of a surface on an incident acoustic wave are typically described by the following parameters: surface impedance (or admittance), reflection coefficient, transmission coefficient and absorption coefficient. These parameters are traditionally defined in the frequency domain and, depending on the context, are used in either frequency and time domains. The first two relate to sound pressure and are complex-valued quantities, while the second two relate to sound energy and are real-valued quantities.

Surface impedance,  $Z$ , is defined as the ratio of sound pressure,  $p$ , and the normal component of particle velocity,  $V_n$ , at the surface. In the frequency domain, the real part of  $Z$  is called resistance and its imaginary part is called reactance:

$$Z(\omega) = R(\omega) + iX(\omega) \quad (1)$$

In this work, we mainly focus on surface impedance. See [1] for the relations among  $Z$  and the other parameters.

## 3 Physical Restrictions on the Impedance Function

When working in the frequency domain, plane-wave reflection of harmonic sound from a uniform surface can be described using a complex number for each angle of incidence.

However, for an incident wave with an extended frequency spectrum (e.g. an impulse), one needs to know the impedance as a function of frequency. This information can be used to translate the impedance into an equivalent function that can be used in the time domain. Time-domain analysis is used in auralization [2] and study of flow ducts [3], for example, and recently in low-frequency room acoustics as well [4].

To extend the definition of impedance, one needs to introduce certain conditions on the impedance function. These conditions ensure that the wall does not violate the conservation of energy, that physical parameters (pressure and velocity) remain real-valued quantities, and that the problem satisfies the principle of causality.

### 3.1 Energy conservation

Conservation of energy requires that the energy of the wave reflected by the wall be smaller than or equal to the energy of the incident wave. In other words, the power reflection coefficient should not exceed unity. This condition requires the surface resistance to be non-negative:  $R(\omega) > 0$ . A surface with this property absorbs sound energy, and is referred to as a passive surface. Note that if a sound source is present on the other side of the wall, and energy is transmitted through the wall, the calculations for conservation of energy are carried out differently.

### 3.2 Reality condition

To ensure that the impedance function remains a real-valued quantity, resistance and reactance,  $R(\omega)$  and  $X(\omega)$ , should be even and odd functions of frequency, respectively. This property is often used to remove negative frequencies from the analysis.

### 3.3 Causality condition

The causality condition states that the pressure at the wall cannot depend on future values of velocity, and vice versa. For this condition to hold,  $Z(\omega)$  should not have any poles or zeros in the upper complex half-space [3]; i.e.  $X(\omega) > 0$ . If the negative Fourier sign convention is used, the upper half-space becomes the lower half-space. Note that violation of the causality condition brings about unphysical instabilities in time-domain analysis [3].

One can show that the real and imaginary parts of a causal impedance function are related by the Hilbert transform [5]. Therefore, knowledge of resistance  $R(\omega)$  is sufficient to completely specify the system, and reactance  $X(\omega)$  carries redundant information. The underlying assumption here is that the system is linear and causal.

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## 4 Common simplifying assumptions

### 4.1 Locally-reacting walls

Local reaction of walls is perhaps the most widely used simplifying assumption used in room acoustics to describe the properties of surfaces. The impedance of a locally-reacting surface is independent of the direction of the incident sound. This is equivalent to assuming that the particle velocity generated by the incident sound at any point on the surface is related linearly to the local pressure only, and is therefore independent of the form of the incident sound field [1]. A surface that is not locally-reacting exhibits extended reaction. A special case of extended reaction is a surface with a reflection coefficient that does not depend on the angle of incident sound. Thus, the walls of an ideal anechoic chamber have extended reaction to the sound field inside the chamber.

Local reaction is encountered wherever the wall itself or the space behind it is unable to propagate waves in a direction parallel to its surface [1]. In room acoustics, walls are not locally-reacting at relatively low frequencies [2]. In particular, modelling multilayer surfaces as locally reacting can make audible mistakes in prediction of parameters that describe the subjective acoustical characteristics of rooms [6, 7]. In general, the assumption of local reaction gives acceptable results in the frequency range in which the sound field may be assumed to be diffuse. It also increases the range of applicability of the data collected from an impedance tube test.

### 4.2 Minimum-phase condition

An assumption that greatly simplifies the relation between the acoustical descriptors of surfaces is the minimum-phase condition. For a minimum-phase function, its magnitude and phase are related through a Hilbert transform [8]. An important implication here is that by measuring the real-valued absorption coefficient of a surface, one can obtain the complex-valued reflection coefficient, and thereby the impedance, of the surface. One of the early applications of the minimum-phase condition in room acoustics is auralization of echogram obtained from ray-tracing by Kuttruff [9].

Interestingly, a surface with a minimum-phase reflection coefficient has the property of reflecting the acoustic energy faster than a surface with the same magnitude of reflection coefficient but a non-minimum phase [8]. It is noted, however, that neither the reflection coefficient nor the transfer function of a room are necessarily minimum phase.

## 5 Concluding Remarks

We gave an overview of some of the theoretical aspects of modelling homogeneous planar surfaces in room acoustics. In particular, we focused on physical restrictions and common simplifying assumptions applied to the surface impedance function. These considerations are particularly important for auralization [2], time-domain simulations of sound fields [4] and improvements of existing prediction models (such as Miki's correction of the Delany-Bazley model [10]). Other aspects of modelling that warrant more research include (i) a

detailed study of the minimum-phase condition in relation to the room transfer function and surface properties; (ii) influence of large objects on the acoustics of a room; (iii) sound fields in rooms coupled by a common wall. An aspect of modelling that has recently received attention is the effects of the finite size of the boundaries [11]. It is worth noting that many of these features are significant for low-frequency acoustical modelling of rooms (up to the Schröder frequency). We hope that this discussion ignites interest and further research on these topics.

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