1 Introduction

Propagation of waves in elastic porous media, e.g. polymeric foams, is described by Biot-Allard’s theory [1]. Two classes of characteristic parameters are needed to describe the porous media in this theory. First, non-acoustic parameters: porosity $\phi$, thermal characteristic length $\Lambda$, flow resistivity $\sigma$, and tortuosity $\alpha_\infty$ which are used in Johnson-Champoux-Allard (JCA) semi-phenomenological model [1]. Second, mechanical parameters, which in the case of isotropic material, are: bulk density $\rho$, Young’s modulus $E$, loss factor $\eta$, and Poisson coefficient $\nu$. The mechanical and non-acoustical properties are inherently dependent on the micro-structure of these materials. But, the internal structure of most porous material, e.g. polyurethane (PU) foam, is too complicated to be studied quantitatively. Therefore, the lattice of PU foams with low relative density ($\rho_r$) is commonly idealized by a tetrakaidecahedral, periodic unit cell (called PUC) and the macroscopic behavior recovered from a dedicated micro-macro approach. Furthermore, the cell windows can be randomly or partially closed, and cells are elongated in the rise direction. Measurements of such lattice results in variability in micro-structure properties of PUC. Hence, a clear understanding of the impact of variability associated with micro-structure properties measurement, and macroscopic Biot’s parameters on vibro-acoustical performance of PU foams is of great importance in the design and optimization of such foams.

The impact of microstructure variability on JCA non-acoustical parameters and sound absorption was investigated by Doutres et. al [2]. This paper build on this work by investigating (i) the effect of the microstructure variability on the elastic properties of the foams and (ii) on the transmission loss of the foam when coupled with an elastic system (a double wall system is studied here). In this regard, a global sensitivity analysis (the FAST method) is applied to the micro-macro based models presented by Doutres et. al [3] and Gholami et. al [4]. The former model links the micro-structure properties of PUC (thickness and length of struts, and the closed windows content) of polyurethane foams to their non-acoustical parameters of JCA model. While, the latter model correlates the micro-structure properties and membrane thickness of PU foam to the Young’s modulus in Biot-Allard’s model.

2 Methodology

2.1 The micro-macro model

The microstructure and non-acoustical properties of PU foams are linked using microstructural based models [3]. Gong’s [5] analytical correlation between Young’s modulus and micro-structure of fully reticulated PU foam was modified by Gholami et. al [4], numerically, to add the effect of closed pore content and membrane thickness on the Young’s modulus:

$$\left( \frac{E}{E_s} \right)_{R_w} = C_E (1 - R_w)^{1.4} + \left( \frac{E}{E_s} \right)_{R_w=1}.$$  \hspace{1cm} (1)

where $C_E = 0.0017$ when membrane thickness is $2.1 \mu m$, and $(E/E_s)_{R_w=1}$ is the relative Young’s modulus of fully reticulated PU foam [5].

2.2 Sensitivity Analysis Method

The contribution of input parameters on the output, in variance based techniques, is investigated by quantifying the impact of input variation on the output variance. A global sensitivity analysis method is used to determine the output sensitivity when the inputs vary over wide ranges. The FAST method is an efficient technique to explore the n-dimensional space of inputs [6]. FAST is used to estimate the first sensitivity index $ST(v)$ (which is the main effect of parameter $v$), and to calculate the total sensitivity index $TSI$ which is defined as the sum of all $SIs$. The ratio of standard deviation to the mean of output, namely Normalized Standard Deviation (NSD) is an efficient indicator that shows the level of variability of the feature of interest [6].

2.3 Description of vibro-acoustic systems

As shown in Fig. 1, $15mm$ PU foam is bonded to the excited, $1mm$ steel plate. The foam is relaxed from a $2mm$ heavy layer (septum) by adding a $5mm$ air space. Decoupling the heavy mass from PU foam by air-gap allows us to account, at the same time, for absorption and isolation problem. Laterally infinite considered system is excited acoustically by an oblique plane wave. The transmission loss is calculated using transfer matrix method [1].

3 Results

3.1 Variability in PUC and Biot’a properties

Two studies are performed in this section. First, the impact of variability in PUC properties of two representative foams, V1 and V2 (see Table 1), are studied. V1 is considered to present a highly reticulated foams and V2 contains a low content of...
Figure 1: Vibro-acoustic configuration of double-wall system.

opened windows. The variability in the PUC is set to 10%. In the second study, the impact of variability in Biot-Allard’s

<table>
<thead>
<tr>
<th>Foam</th>
<th>$t \times 10^{-6} (m)$</th>
<th>$l \times 10^{-6} (m)$</th>
<th>$R_w (%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>51.3, 62.7</td>
<td>[157.9, 192.5]</td>
<td>70, 90</td>
</tr>
<tr>
<td>V2</td>
<td>51.3, 62.7</td>
<td>[157.9, 192.5]</td>
<td>10, 30</td>
</tr>
</tbody>
</table>

Table 1: Inferior and superior limits for microstructure properties.

Table 2: Inferior and superior limits for Biot’s properties.

of V1 and V2 are shown in Fig. 2. It is shown that strut length is main parameter for V1. While, the reticulation rate is the dominant parameter for V2 mainly at double wall frequency, frame born resonance and at high frequency in the cavity absorption region. The contribution of macroscopic parameters of vibro-acoustic system on TL are shown in Fig. 3. As expected the mass of plate and septum are the dominant parameter at low frequency. However, tortuosity and flow resistivity are dominant at high frequency in the cavity absorption controlled region where the performance of the foam is important. Mechanical properties of the foam are important at the frame resonance of of the foam.

4 Conclusions

The impacts of PU foam microstructure variability and macroscopic properties on the TL of a laterally infinite double wall system are studied. It is shown that the impact of strut length is dominant where foam is highly reticulated. While, the reticulation rate takes the lead when close pore content is high. Therefore, measuring precisely these parameters helps to reduce uncertainty in TL prediction of such a system. It should be mentioned that, for macroscopic properties, septum mass and plate are important at low frequencies, mechanical properties of the foam are dominant at foam resonance and non acoustical properties are important at high frequency.

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References


