

# BAYESIAN INVERSION OF TIME-DIFFERENCE-OF-ARRIVAL DATA TO LOCALIZE BOWHEAD WHALES IN THE CHUKCHI SEA

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## 1 Introduction

Localizing marine mammals without disrupting their behaviour is often performed using passive acoustic monitoring techniques. Passive localization over large areas requires widely-spaced acoustic recorders, which often precludes inter-recorder clock synchronization and applications of well-established matched-field or time-difference-of-arrival (TDOA) localization techniques. In this paper, we localize bowhead whales in the northeastern Chukchi Sea using a nonlinear Bayesian inversion of whale-call TDOA data obtained from non-synchronized recorders. The inversion is applied to batches of TDOA data in which the whale locations, relative recorder clock drifts, effective environmental sound speed, and recorder locations are all treated as unknown parameters (subject to prior information) and estimated in the inversion. Rigorous posterior uncertainty estimates for the parameters are provided by the Bayesian formulation. A more complete treatment of this work can be found in [1].

## 2 Bowhead-Whale Call Data

JASCO Applied Sciences recorded thousands of low-frequency bowhead-whale calls in the Chukchi Sea on a number of Autonomous Multichannel Acoustic Recorders (AMARs) during August–October, 2013 [2]. Although the AMAR internal clocks were synchronized at deployment, the clocks drift out of synchronization relative to each (a common problem for long-term instrument deployments). This paper considers whale calls recorded on up to seven AMARs (herein labelled A–G) over a 30 minute period on 11 October. The recordings were initially aligned roughly by observing similar time-frequency characteristics of transient sounds (e.g., seismic pulses or specific bowhead-whale calls). Whale calls were then manually associated between recorders by listening to the recording and visually comparing call spectrograms. Each call annotation defined lower and upper frequencies and start and end call times. TDOA data were then derived according to Sec. 3.1.

## 3 Methods

### 3.1 Data Processing

For each bowhead-whale call, recordings were band-pass filtered according to the frequency annotations and cross-correlated with all other recordings of the same call. TDOA data were determined from the maximum of the cross-correlation functions, providing time delays relative to an

arbitrarily-assigned reference recorder. In this manner, each whale call yielded up to 21 TDOAs, which were ranked in descending order according to the the maximum value of the cross-correlation function (a measure of signal similarity). A minimum of 2 and up to 6 linearly-independent TODAs were selected for the inversion (linearly-dependent TDOAs were discarded for consistency with the assumption of independent data errors in Sec. 3.3).

### 3.2 Bayesian Inversion

Whale-call TDOA data depend on the whale and recorder locations, relative recorder clock drifts, and the environmental sound speed. Using a straight-path, time-of-flight propagation model, the TDOA for call  $w$  between recorders  $i$  and  $j$  is

$$t_{wi} - t_{wj} = \frac{\sqrt{(x_w - X_i)^2 + (y_w - Y_i)^2}}{c} - \frac{\sqrt{(x_w - X_j)^2 + (y_w - Y_j)^2}}{c} + \Delta_i - \Delta_j, \quad (1)$$

where  $x_w$  and  $y_w$  are the easting and northing coordinates of the whale,  $X_i$  and  $Y_i$  are the coordinates of the  $i$ th recorder,  $c$  is the effective waveguide sound speed (discussed below), and  $\Delta_i$  and  $\Delta_j$  are the recorder clock drifts relative to the reference recorder (this formulation assumes the relative clock drifts remain constant between the first and last whale calls in the data batch). Since the bowhead-whale calls propagate as normal modes in the shallow-water environment ( $\sim 40$  m depth), the sound speed  $c$  is an effective average modal group speed (less than the water sound speed) weighted by the frequency and modal content of the calls. The recorder locations are also considered unknown parameters in the inversion, subject to Gaussian priors centred at the deployment locations with 50 m standard deviations to account for uncertainties in GPS ship locations and lateral drift of the AMARS as they are lowered through the water. This is done so the whale-location uncertainties account for recorder-location uncertainties; high-precision recorder localization is not expected. In the Bayesian formulation the solution consists of properties of the posterior probability density (PPD) of the unknown parameters given the measured data and prior information. The PPD is approximated numerically for non-linear problems using a Markov-chain Monte Carlo algorithm [3].

### 3.3 Likelihood

Residual errors between measured and predicted data are assumed to be uncorrelated and Gaussian-distributed within a batch of whale calls, with an unknown standard deviation  $\sigma$

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treated as a nuisance parameter in the inversion. For  $N$  data the corresponding likelihood function is

$$L(\mathbf{m}) = \frac{1}{(2\pi)^{N/2}\sigma^N} \exp\left[-\frac{|\mathbf{d} - \mathbf{d}(\mathbf{m})|^2}{2\sigma^2}\right], \quad (2)$$

where  $\mathbf{m}$  is the vector of unknown model parameters (whale and recorder locations, relative recorder clock drifts, and sound speed), and  $\mathbf{d}$  and  $\mathbf{d}(\mathbf{m})$  are measured and predicted data, respectively. Setting  $\partial L/\partial\sigma = 0$  leads to a maximum-likelihood estimate for the error standard deviation  $\sigma$ :

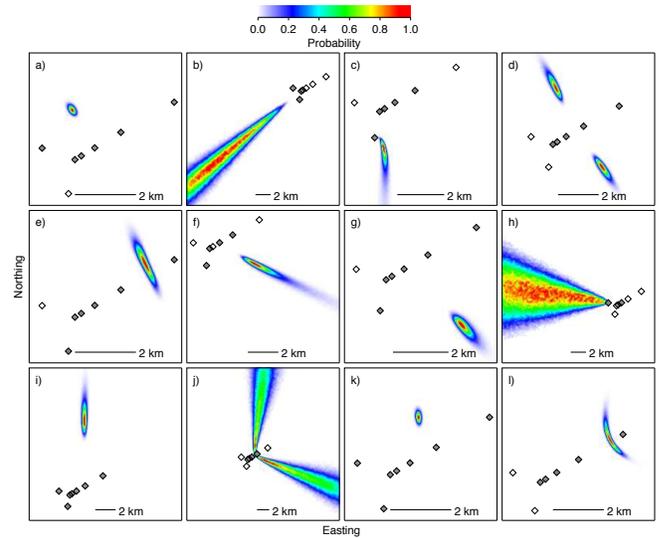
$$\hat{\sigma}(\mathbf{m}) = [|\mathbf{d} - \mathbf{d}(\mathbf{m})|^2/N]^{1/2}. \quad (3)$$

Equation (3) allows implicit sampling of  $\hat{\sigma}$  by sampling explicitly over the parameters in  $\mathbf{m}$ .

## 4 Inversion Results

Figure 1 shows examples of the marginal probability densities for bowhead-whale locations in the horizontal plane computed for 12 whale calls via the Bayesian inversion algorithm. This figure illustrates the variety of uncertainty distributions obtained, which are strongly dependent on the source-recorder geometry for the specific case in each panel (note that not all recorders detected all calls). Localization uncertainties generally increase with the whale’s distance from the centre of the AMAR cluster. When the off-axis recorders do not detect a call, the probability densities are symmetric about the cluster axis, as shown in Fig. 1(d), (j), and (l). Further, the marginal densities in Fig. 1(d) and (j) are multi-modal, and the marginal in Fig. 1(j) is notably curved (non-Gaussian). These three marginal PPDs illustrate the potentially strong non-linearity of the localization problem, and could not be recovered by a linearized inversion algorithm. Resolving such probability densities with the Bayesian sampling algorithm required parallel tempering [4] to provide sufficiently wide, but efficient, sampling. Call locations oriented broadside to the axis of the AMAR array are generally well localized, with the highest precision for calls that are also relatively close to the array (e.g., Fig. 1(a) and (k)). Endfire call locations are poorly constrained in terms of range to the AMAR cluster; however, the direction of arrival is well determined (e.g., Fig. 1(b) and (h)).

In terms of the other parameters included in the inversion, the effective sound speed was estimated at  $1403 \pm 18$  m/s (one standard-deviation uncertainty), which corresponds approximately to the average group speed of the first two modes computed for the shallow-water environment at the whale-call frequencies. Relative clock drifts were estimated to uncertainties of generally less than 50 ms, and an essentially linear clock-drift trend over a two-week period was determined by considering multiple TODA data sets (not shown here). The posterior localization uncertainties for the recorders were similar to their prior uncertainties indicating the inversion did not significantly improve these locations; however, this procedure accounts for the recorder-location uncertainties in the whale-location uncertainty estimates. The estimated data standard deviations varied from 0.10–0.14 s, indicating the TODA data are well fit by the inversion.



**Figure 1:** Posterior marginal probability densities for bowhead whale locations for 12 selected calls. Prior recorder (AMAR) locations (GPS deployment positions) are shown with diamond symbols; filled diamonds indicate recorders which detected the call considered in that panel, open diamonds represent recorders which did not detect the call.

## 5 Summary

This paper presented a nonlinear Bayesian inversion to passively localize bowhead whales in the Chukchi sea based on TODA data from an array of autonomous, non-synchronized recorders. The inversion rigorously quantifies localization uncertainties, accounting for unknown recorder clock drifts and for uncertainties in the TODA data, recorder locations, and effective environmental sound speed. Localization uncertainties are found to be strongly dependent on the particular source-recorder geometries with strong nonlinear effects.

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## References

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